Antenna Design Supplement

LAMBDA FUNCTIONS DESCRIBE ANTENNA/DIFFRACTION PATTERNS

Antenna pattern design is simplified by unifying functions which reduce pattern integration effort. It turns out that nearly all the familiar patterns from linear, rectangular and circular apertures belong to a common family mathematically expressed by the Lambda functions given here. Twenty-three pages of curves, standard tables and formulas provide a permanent reference for the antenna engineer.

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Lambda Functions

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Beam antenna theory had its beginnings with the introduction of the HF beam-forming arrays in the early 1920's. Yet it is still difficult to locate good plots of standard radiation patterns. Textbooks and manuals have tended to avoid the cost of expensive pattern plots. Articles usually treat limited types. In this supplement, we have collected some basic patterns of antenna theory and plotted them as readable graphs from which beam and sidelobe amplitudes can be obtained with sufficient accuracy for most engineering purposes.

Application of the patterns

The aperture illuminations and radiation patterns given in the supplement constitute a selection of basic Fourier transform graphs, of value in antenna design. The utility of the patterns extends, however, into other fields of science and engineering. Thus, the antenna designer who designs antennas for communications, radar, navigational aids, radio-astronomy, propagation and scattering studies, and so on, will find his system colleagues also can use these patterns in other "domains." Most commonly, the electrical signal in time replaces the aperture illumination and the frequency spectrum replaces the antenna pattern, so that the signals engineer works in the "time-frequency" domain instead of the "aperture-space" domain.

Of course, antenna patterns are diffraction patterns, so that optical and acoustic use of the patterns is immediate. Microwave engineers who design directional couplers, transversal filters, inhomogeneous lines, and tubes also encounter the basic Fourier relationship. In mechanics and elasticity, the vibrations of strings, membranes, cylinders and spheres, and the oscillations of chains and beams can be characterized by Fourier transformations. Diffusion phenomena, whether thermal or neutron, hydrodynamical flow through apertures, surface waves, and many other phenomena of physics also can be treated by Fourier integrals.

Several branches of mathematics make use of Fourier transforms. Curiously, the Lambda functions and transforms have not had the attention they deserve. This supplement illustrates how useful these mathematical tools can be to the antenna engineer.

Explanation of patterns

Antenna patterns of sharp beam antennas are Fourier transforms of the aperture excitation. In the historical evolution of directive antennas, the preferred aperture shapes have been either rectangular or circular, with elliptical falling between. In the supplement, we deal with rectangular and circular apertures; but because rectangular apertures usually have separable illuminations in perpendicular directions, it suffices to treat linear and circular apertures. Fortunately, these basic shapes have a common theory because each has only one coordinate if the circular aperture is symmetrically illuminated. It turns out that all the basic antenna patterns of linear or circular apertures are described by Lambda Functions.

A brief list of the Lambda functions is given in Table 1. The plotted patterns of Figs. 1-20 are particular cases of Lambda functions. Computer print-outs of these valuable functions are given in Tables 2 and 3.

The graphs show the aperture illumination and the accompanying radiation pattern for different types of aperture illumination, old and new. In addition, some Fresnel-region patterns are included; these patterns are also far-field patterns of apertures with quadratic phase error.

The patterns plotted are field strength patterns and are mainly normalized. The field angle, θ, is contained in the variable $u = \sin\theta/\lambda$ which experience has shown is the best Fourier variable for the purpose. Linear apertures have aperture width a, and circular apertures diameter D, so that the pattern variable is πau or πDu, which the user can write as α or β if he desires. He is advised not to shorten the notation because π plays a fundamental role relating to the position of the nulls in patterns. It is better to express the generalized angle in π's than in radians.

Patterns of linear and circular apertures are plotted together on the same page, the linear on the left side of the center axis, the circular on the right. Beamwidths at the -3 dB level are given; other beam widths can be obtained from the curves. Peak sidelobe levels are marked in dB. The defining functions of the illuminations and patterns are given on each graphed transform pair.

When opposite halves of a symmetrically illuminated linear or circular aperture are in antiphase, the patterns are usually described by "Lambda-Struve" functions, an important class of odd functions which are as naturally suited to the antiphase illuminations as the Lambda functions of the first kind are to the equiphase illuminations. Some antiphase Lambda-Struve patterns are also plotted in the accompanying figures.

Fresnel-region patterns of linear radiatiors and of circular apertures can be treated together by means of the Lommel functions. Examples are given for uniform illumination only due to limitations of space.

A short account of the unifying Lambda theory of far-field patterns is provided in the following section, with a note on the unifying Lommel theory of Fresnel region patterns.

Unified theory of apertures

The radiation pattern $E_L(u)$ of an equiphase linear radiator having a symmetrical illumination function $F(x)$ is the Fourier cosine transform of $F(x)$; that is

$$E_L(u) = 2\int_0^{a/2} F(x)\cos 2\pi ux\,dx, \tag{1}$$

where the origin is at the center of the linear radiator of aperture width a, and $u = (\sin\theta)/\lambda$, the angle θ being measured from the normal to the aperture in a plane containing the linear radiator.

The radiation pattern $E_c(u)$ of an equiphase circular aperature having a circularly symmetrical illumination function $F(\rho)$ is the zero-order Fourier-Bessel or Hankel transform,

$$E_C(u) = 2\pi\int_0^{D/2} F(\rho)\,J_0(2\pi u\rho)\,\rho\,d\rho \tag{2}$$

where ρ is the radial coordinate of the circular aperture of diameter D, and $u = (\sin\theta)/\lambda$. The angle θ is measured from the normal to the circular radiator in any plane containing the normal.

The above two Fourier transforms are particular cases of the Lambda transform

$$E_\nu(u) = \frac{2\pi^{\nu+1}}{\Gamma(\nu+1)}\int_0^{\alpha/2} F(x)\,\Lambda_\nu(2\pi ux)\,x^{2\nu+1}\,dx. \tag{3}$$

The Lambda function of the first kind is defined in terms of the Bessel function of the first kind of order ν by

$$\Lambda_\nu(z) = \frac{\Gamma(\nu+1)}{(z/2)^\nu}J_\nu(z) \tag{4}$$

and $\Gamma(\nu+1) = \nu\Gamma(\nu)$, with $\Gamma(1/2) = \sqrt{\pi}$, is the Gamma or factorial function.

The Fourier cosine transform of Eq. 1 appears from Eq. 3 when $\nu = -1/2$, for then,

$$\Lambda_{-1/2}(2\pi ux) = \cos 2\pi ux \tag{5}$$

and we take $\alpha = a$ for a linear antenna.

The Hankel transform of Eq. 2 appears when $\nu = 0$, for then,

$$\Lambda_0(2\pi ux) = J_0(2\pi ux) \tag{6}$$

with $\alpha = D$ for a circular antenna.

Thus, if $E(u)$ is calculated by Eq. 3, then

$$E_L(u) = E_{-1/2}(u) \text{ and } E_c(u) = E_0(u). \tag{7}$$

Therefore, separate integrations for the patterns of a linear aperture and of a circular aperture with the illumination $F(x)$ are not necessary. Both patterns are obtained by the single integration of Eq. 3, yielding the "Lambda pattern," $E_\nu(u)$.

If the illumination is uniform, $F(x) = 1$, the Lambda pattern is

$$E_\nu(u) = \left(\frac{\pi\alpha^2}{4}\right)^{\nu+1}\frac{\Lambda_{\nu+1}(\pi\alpha u)}{\Gamma(\nu+2)}. \tag{8}$$

If $\nu = -1/2$, the pattern is that of a linear radiator:

$$E_L(u) = a\Lambda_{\frac{1}{2}}(\pi au) = a\frac{\sin \pi au}{\pi au}. \tag{9}$$

If $\nu = 0$, the pattern is that of a circular aperture:

$$E_C(u) = \frac{\pi D^2}{4}\Lambda_1(\pi Du) = \frac{\pi D^2}{4}\frac{2J_1(\pi Du)}{\pi Du}. \tag{10}$$

These classical optical diffraction patterns are plotted in Fig. 1.

If the equiphase illumination is a member of the inverted parabola family, linear and circular aperture patterns are derived from the corresponding Lambda patterns. The Lambda pattern formula is given in Table 1 and the particular patterns are plotted and summarized in Figs. 2 and 4.

Cosine and cosine squared functions are traditionally used as illuminations for linear radiators. The analogous illuminations for a circular aperture are: (1) the Bessel function of zero order taken to its first zero; and (2) the same function taken to its first minimum and raised on a pedestal to yield zero amplitude at the periphery of the aperture. All these trigonometrical and Bessel illuminations involve Lambda functions of order ν. A Lambda pattern then exists for the basic cosine/Bessel pair from which many particular patterns are obtained. Table 1 gives the general Lambda pattern and Figs. 5-10 illustrate the resulting linear and circular aperture patterns.

Both the inverted parabola and the Lambda illuminations show low near-in sidelobes for special values of the edge taper. It is of interest to note that the illumination

$$F(\rho) = \frac{1}{3} + \frac{2}{3}J_0\left(7.66\frac{\rho}{D}\right) \tag{11}$$

is a low-sidelobe weighting for a circular aperture analogous to the Hamming weighting for a linear radiator (see Fig. 10).

Taylor, Bickmore, and Spellmire have established the value of an integral due to Sonine in yielding controllable patterns of linear radiators. A Lambda version of Sonine's integral extends the approach to include the circular aperture. The resulting three parameter Lambda patterns are given in Table 1, and include the patterns for uniform illumination and for the parabolic family. Examples of particular patterns and summary data are shown in Figs. 12-15.

If separate halves of either a linear radiator or a circular aperture are in relative antiphase, an antiphase Lambda pattern yields the patterns of both types of antenna. The generalized antiphase pattern is given by

$$\overline{E}_\nu(u) = \frac{2\pi^{\nu+1}}{\Gamma(\nu+1)} \int_0^{\alpha/2} F(x)\, \Lambda H_\nu(2\pi u x)\, x^{2\nu+1}\, dx, \tag{12}$$

where $F(x)$ is now an odd function on a linear radiator, or is odd in the plane of antiphase for a circular aperture which is symmetrically illuminated in intensity. The Lambda-Struve function in the integrand is defined by

$$\Lambda H_\nu(z) = \frac{\Gamma(\nu+1)}{(z/2)^\nu} H_\nu(z), \tag{13}$$

where $H_\nu(z)$ is the Struve function.

Antiphase uniform illumination (signum x) has a Lambda-Struve pattern

$$\overline{E}_\nu(u) = \left(\frac{\pi\alpha^2}{4}\right)^{\nu-1} \frac{\Lambda H_{\nu+1}(\pi\alpha u)}{\Gamma(\nu+2)}. \tag{14}$$

A linear aperture has then a pattern given by $\nu = -1/2$:

$$\overline{E}_L(u) = a\Lambda H_{\frac{1}{2}}(\pi a u) = a\,\frac{1-\cos\pi a u}{\pi a u} \tag{15}$$

A circular aperture has the pattern

$$\overline{E}_c(u) = \frac{\pi D^2}{4} \Lambda H_1(\pi D u) = \frac{\pi D^2}{4} \frac{2H_1(\pi D u)}{\pi D u}. \tag{16}$$

These antiphase patterns and antiphase for parabolic illuminations are plotted in Figs. 17 and 18. Note that the Struve and Lambda-Struve functions are never negative if u is positive.

Fresnel region patterns of linear, rectangular and circular apertures are unified by means of Lommel functions. If the illumination is uniform, the general Fresnel field strength pattern is given by

$$E_\nu(w,z) = U_{\nu+1}(w,z) + j\,U_{\nu+2}(w,z), \tag{17}$$

$$\text{where } w = \frac{\pi\alpha^2}{2\lambda R} = \pi\frac{R_r}{R}$$

and

$$\alpha \begin{cases} = a \text{ for a line source,} \\ = D \text{ for a circular aperture,} \end{cases}$$

R = range, λ = wavelength,
R_r = Rayleigh range $= \alpha^2/2\lambda$,
$z = \pi\alpha u$, $u = (\sin\theta)/\lambda$.

The Lommel function is defined by

$$U_\nu(w,z) = \sum_{m=0}^{\infty} (-)^m (w/z)^{\nu+2m} J_{\nu+2m}(z), \tag{18}$$

The Fresnel region pattern of a line source is obtained by letting $\nu = -1/2$. Then,

$$E_L(w,z) = U_{1/2}(w,\pi a u) + j\,U_{3/2}(w,\pi a u). \tag{19}$$

The Fresnel pattern of a circular aperture with $\nu = 0$ is

$$E_c(w,z) = U_1(w,\pi D u) + j\,U_2(w,\pi D u). \tag{20}$$

Examples of the modulus of these patterns are shown in Figs. 19 and 20.

The unified Lommel approach can be extended to tapered illuminations of linear radiators and circular apertures. Although the Fresnel patterns of linear radiators can be expressed in Fresnel integrals, the use of the Lommel functions $U_{1/2}(w,z)$ and $U_{3/2}(w,z)$ leads to more compact expressions. • •

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Bibliography

1. Bickmore, R. W. and Spellmire, R. J. "A Two-Parameter Family of Line Sources," Hughes Aircraft Company, Culver City, California, Technical Memo. No. 595, October 1956.
2. Hamming, R. W., See "The Measurement of Power Spectra" by R. B. Blackmore and J. W. Tukey, Dover Publications, New York, 1958.
3. Hansen, R. C., "Aperture Theory." Chapter I of "Microwave Scanning Antennas" Vol. I. (Ed: R. C. Hansen) Academic Press, New York, 1964. Excellent survey. Good bibliography.
4. Ishimaru, A. and Held, G. "Analysis and Synthesis of Radiation Patterns from Circular Apertures." Canadian J. Physics, Vol. 38, 78-99, 1960. Includes Bessel illuminations.
5. Ling, C., Lefferts, E., Lee, D., and Potenza, D. "Radiation Patterns of Planar Antennas with Optimum and Arbitrary Illuminations," IEEE. International Convention Record, 1964. Rectangular and circular apertures. Cosine and Bessel illuminations.
6. Ramsay, J. F. (on Lambda functions and applications)
   a. "Introduction to Lambda Functions and Transforms," Symposium Digest, IEEE 1963 PGAP International Symposium, Boulder, Colo.
   b. "Lambda Functions in Antenna Theory," Symposium Digest, IEEE 1964 PGAP International Symposium, Kennedy Airport, New York.
   c. "Rayleigh Distance as a Normalizing Range for Beam Power Transmission," Symposium Digest, IEEE 1965 G-MTT Symposium, Clearwater, Florida.
   d. "Lambda Functions for Dipole Applications," IEEE 1966 International Convention Record, Pt 5, 186-199.
   e. "Tubular Beams from Radiating Apertures." In "Advances in Microwaves," Vol. III. (Ed. Leo Young), Academic Press, New York. To be published.
7. Sciambi, A. F., Jr. "Instant Antenna Patterns," MicroWaves, Vol. 5, 48-60, June 1966. Circular aperture. Inverted parabola family with pedestals.
8. Spencer, R. C. "Paraboloid Diffraction Patterns from the Standpoint of Physical Optics," MIT Radiation Laboratory Report T-7, October 21, 1942. Fourier Transforms. Lambda Functions attributed to L. J. Chu. A classic, still obtainable. See also references to Spencer's reports in MIT Series Vol. 12 (Ed: S. Silver)
9. Taylor, T. T. "One Parameter Family of Line Sources having modified sin $\pi u/\pi u$ Patterns", Hughes Aircraft Company, Culver City, California, Technical Memo. No. 324, September 4, 1953.
10. White, W. D., "Pattern Limitations in Multiple Beam Antennas." I.R.E. Trans. Ant. Prop., Vol. AP-10, 430-436, July 1962. Linear aperture. Cosine varieties, including pedestals.

Table 1. Lambda function formulas

LAMBDA FUNCTIONS DEFINED

$$\Lambda_\nu(x) = \frac{\Gamma(\nu+1)}{(x/2)^\nu} J_\nu(x) \qquad \Lambda H_\nu(x) = \frac{\Gamma(\nu+1)}{(x/2)^\nu} H_\nu(x)$$

ELEMENTARY LAMBDA FUNCTIONS

EVEN FUNCTIONS	ODD FUNCTIONS
$\Lambda_{-1/2}(x) = \cos x$	$\Lambda H_{-1/2}(x) = \sin x$
$\Lambda_0(x) = J_0(x)$	$\Lambda H_0(x) = H_0(x)$
$\Lambda_{1/2}(x) = \frac{\sin x}{x}$	$\Lambda H_{1/2}(x) = \frac{1-\cos x}{x}$
$\Lambda_1(x) = \frac{2J_1(x)}{x}$	$\Lambda H_1(x) = \frac{2H_1(x)}{x}$
$\Lambda_{3/2}(x) = \frac{3}{x^2}\left(\frac{\sin x}{x} - \cos x\right)$	$\Lambda H_{3/2}(x) = -\frac{3}{x^2}\left(\frac{\cos x}{x} + \sin x - \frac{1}{x} - \frac{x}{2}\right)$
$\Lambda_2(x) = \frac{8J_2(x)}{x^2}$	$\Lambda H_2(x) = \frac{8H_2(x)}{x^2}$

RECURRENCE RELATIONS

$$\Lambda_{\nu+1}(x) = \frac{\nu(\nu+1)}{(x/2)^2}\left[\Lambda_\nu(x) - \Lambda_{\nu-1}(x)\right] \qquad \frac{\Lambda_{\nu+1}(x)}{\nu+1} = -\frac{2}{x}\frac{d}{dx}\Lambda_\nu(x)$$

$$\Lambda H_{\nu+1}(x) = \frac{\nu(\nu+1)}{(x/2)^2}\left[\Lambda H_\nu(x) - \Lambda H_{\nu-1}(x)\right] + \frac{2}{x}\frac{\Gamma(\nu+2)}{\sqrt{\pi}\,\Gamma(\nu+3/2)}$$

$$\frac{\Lambda H_{\nu+1}(x)}{\nu+1} = -\frac{2}{x}\frac{d}{dx}\Lambda H_\nu(x) + \frac{2}{x}\frac{\Gamma(\nu+1)}{\sqrt{\pi}\,\Gamma(\nu+3/2)}$$

LAMBDA RADIATION PATTERNS (FOR LINEAR RADIATOR, PUT $\nu = -1/2$; FOR CIRCULAR APERTURE, PUT $\nu = 0$)

APERTURE ILLUMINATION	RADIATION PATTERN
UNIFORM $F(x)=1$	$\left(\frac{\pi a^2}{4}\right)^{\nu+1}\frac{1}{\Gamma(\nu+2)}\Lambda_{\nu+1}(\pi a u)$
INVERTED PARABOLIC $F(x)=\left[1-\left(\frac{2}{a}x\right)^2\right]^p$	$\left(\frac{\pi a^2}{4}\right)^{\nu+1}\frac{\Gamma(p+1)}{\Gamma(\nu+p+2)}\Lambda_{\nu+p+1}(\pi a u)$
SONINE (INCLUDING TAYLOR MODIFIED TYPE) $F(x)=\left[1-\left(\frac{2}{a}x\right)^2\right]^p \Lambda_p\left[j\pi a u_0\sqrt{1-\left(\frac{2}{a}x\right)^2}\right]$	$\left(\frac{\pi a^2}{4}\right)^{\nu+1}\frac{\Gamma(p+1)}{\Gamma(\nu+p+2)}\Lambda_{\nu+p+1}\left[\sqrt{(\pi a u)^2-(\pi a u_0)^2}\right]$
LAMBDA (COSINE AND BESSEL TYPE $F(x)=\Lambda_\nu\left(\lambda_1 \frac{2}{a}x\right)$	$\left(\frac{\pi a^2}{4}\right)^{\nu+1}\frac{\lambda_1^2}{\Gamma(\nu+2)}\Lambda_{\nu+1}(\lambda_1)\frac{\Lambda_\nu(\pi a u)}{\lambda_1^2-(\pi a u)^2}$

LAMBDA MODIFIED FUNCTION, $\Lambda I_\nu(x) = \Lambda_\nu(jx)$

$$\Lambda I_{-1/2}(x) = \cosh x \qquad \Lambda I_0(x) = I_0(x)$$

$$\Lambda I_{1/2}(x) = \frac{\sinh x}{x} \qquad \Lambda I_1(x) = \frac{2I_1(x)}{x}$$

$$\Lambda I_{3/2}(x) = -\frac{3}{x^2}\left(\frac{\sinh x}{x} - \cosh x\right) \qquad \Lambda I_2(x) = \frac{8I_2(x)}{x^2}$$

$$\Lambda I_{\nu+1}(x) = \frac{\nu(\nu+1)}{(x/2)^2}\left[\Lambda I_{\nu+1}(x) - \Lambda I_\nu(x)\right]$$

$$\frac{\Lambda I_{\nu+1}(x)}{\nu+1} = \frac{2}{x}\frac{d}{dx}\Lambda I_\nu(x)$$

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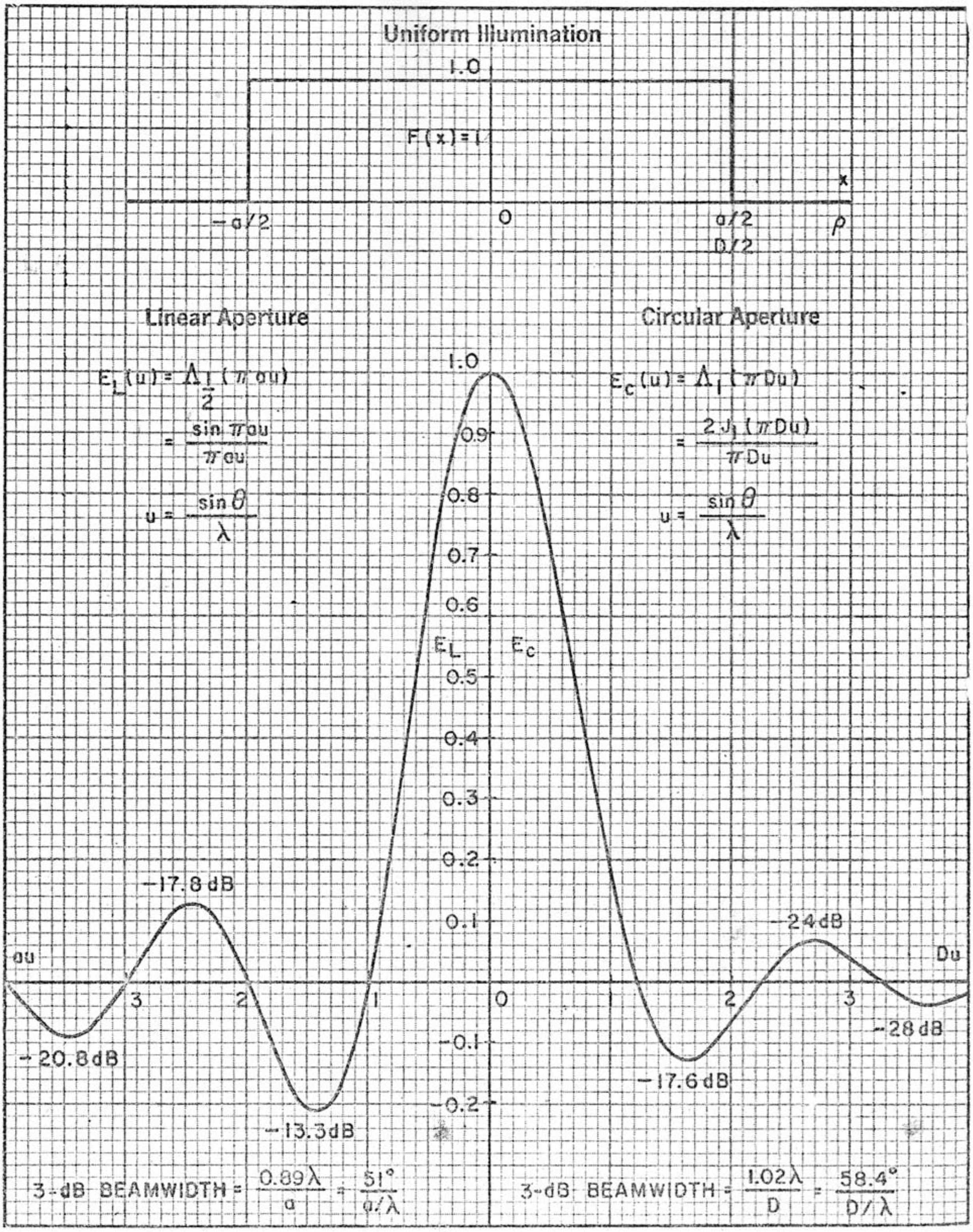
Table 2. Lambda functions

X	$\Lambda_{-\frac{1}{2}}(\pi X)$	$\Lambda_0(\pi X)$	$\Lambda_{\frac{1}{2}}(\pi X)$	$\Lambda_1(\pi X)$	$\Lambda_{\frac{3}{2}}(\pi X)$	$\Lambda_2(\pi X)$
	$\cos \pi X$	$J_0(\pi X)$	$\frac{\sin \pi X}{\pi X}$	$\frac{2J_1(\pi X)}{\pi X}$	SEE TABLE 1	$\frac{8J_2(\pi X)}{(\pi X)^2}$
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
.1	.95106	.97548	.98363	.98771	.99017	.99180
.2	.80902	.90371	.93549	.95146	.96107	.96750
.3	.58779	.78996	.85839	.89300	.91395	.92800
.4	.30902	.64251	.75683	.81518	.85074	.87473
.5	0.00000	.47200	.63662	.72170	.77404	.80960
.6	-.30902	.29056	.50455	.61696	.68693	.73491
.7	-.58779	.11085	.36788	.50574	.59283	.65323
.8	-.80902	-.05496	.23387	.39294	.49531	.56727
.9	-.95106	-.19615	.10929	.28332	.39791	.47980
1.0	-1.00000	-.30424	0.00000	.18119	.30396	.39348
1.1	-.95106	-.37364	-.08942	.09024	.21645	.31075
1.2	-.80902	-.40199	-.15591	.01330	.13786	.23376
1.3	-.58779	-.39027	-.19809	-.04773	.07009	.16429
1.4	-.30902	-.34262	-.21624	-.09197	.01439	.10366
1.5	0.00000	-.26586	-.21221	-.11954	-.02867	.05271
1.6	.30902	-.16886	-.18921	-.13148	-.05916	.01184
1.7	.58779	-.06172	-.15148	-.12961	-.07775	-.01904
1.8	.80902	.04518	-.10394	-.11640	-.08565	-.04042
1.9	.95106	.14198	-.05177	-.09471	-.08444	-.05315
2.0	1.00000	.22028	0.00000	-.06760	-.07597	-.05834
2.1	.95106	.27371	.04684	-.03812	-.06232	-.05732
2.2	.80902	.29848	.08504	-.00909	-.04547	-.05151
2.3	.58779	.29358	.11196	.01707	-.02734	-.04237
2.4	.30902	.26076	.12614	.03846	-.00965	-.03128
2.5	0.00000	.20427	.12732	.05380	.00619	-.01951
2.6	-.30902	.13039	.11643	.06243	.01913	-.00815
2.7	-.58779	.04677	.09538	.06435	.02849	.00195
2.8	-.80902	-.03830	.06682	.06010	.03396	.01017
2.9	-.95106	-.11670	.03392	.05070	.03560	.01613
3.0	-1.00000	-.18121	0.00000	.03750	.03377	.01970
3.1	-.95106	-.22616	-.03173	.02207	.02908	.02094
3.2	-.80902	-.24789	-.05847	.00600	.02228	.02010
3.3	-.58779	-.24506	-.07804	-.00919	.01423	.01756
3.4	-.30902	-.21868	-.08904	-.02220	.00578	.01378
3.5	0.00000	-.17197	-.09095	-.03203	-.00226	.00926
3.6	.30902	-.10998	-.08409	-.03805	-.00922	.00450
3.7	.58779	-.03907	-.06960	-.04001	-.01460	-.00006
3.8	.80902	.03373	-.04974	-.03805	-.01807	-.00403
3.9	.95106	.10137	-.02522	-.03266	-.01951	-.00714
4.0	1.00000	.15751	0.00000	-.02459	-.01900	-.00923
4.1	.95106	.19703	.02399	-.01479	-.01676	-.01021
4.2	.80902	.21654	.04455	-.00428	-.01317	-.01015
4.3	.58779	.21464	.05989	.00591	-.00868	-.00915
4.4	.30902	.19202	.06880	.01485	-.00377	-.00742
4.5	0.00000	.15132	.07074	.02180	.00106	-.00518
4.6	-.30902	.09687	.06581	.02624	.00538	-.00271
4.7	-.58779	.03420	.05479	.02791	.00884	-.00023
4.8	-.80902	-.03045	.03898	.02682	.01119	.00202
4.9	-.95106	-.09083	.02007	.02326	.01229	.00385
5.0	-1.00000	-.14118	0.00000	.01770	.01216	.00515
5.1	-.95106	-.17685	-.01929	.01078	.01089	.00585
5.2	-.80902	-.19469	-.03598	.00323	.00869	.00593
5.3	-.58779	-.19331	-.04859	-.00420	.00583	.00546
5.4	-.30902	-.17321	-.05606	-.01082	.00264	.00451
5.5	0.00000	-.13668	-.05787	-.01606	-.00058	.00323
5.6	.30902	-.08754	-.05406	-.01948	-.00352	.00176
5.7	.58779	-.03079	-.04518	-.02088	-.00592	.00025
5.8	.80902	.02797	-.03226	-.02021	-.00760	-.00116
5.9	.95106	.08301	-.01667	-.01764	-.00845	-.00234
6.0	1.00000	.12906	0.00000	-.01352	-.00844	-.00321
6.1	.95106	.16183	.01613	-.00831	-.00764	-.00371
6.2	.80902	.17835	.03018	-.00255	-.00616	-.00381
6.3	.58779	.17728	.04088	.00317	-.00419	-.00356
6.4	.30902	.15902	.04730	.00833	-.00194	-.00298
6.5	0.00000	.12559	.04897	.01246	.00035	-.00217
6.6	-.30902	.08048	.04587	.01520	.00248	-.00121
6.7	-.58779	.02822	.03844	.01637	.00424	-.00021
6.8	-.80902	-.02600	.02751	.01592	.00550	.00073
6.9	-.95106	-.07691	.01426	.01397	.00616	.00155
7.0	-1.00000	-.11961	0.00000	.01076	.00620	.00216
7.1	-.95106	-.15008	-.01385	.00665	.00565	.00252
7.2	-.80902	-.16553	-.02599	.00208	.00459	.00262
7.3	-.58779	-.16468	-.03528	-.00251	.00315	.00247
7.4	-.30902	-.14783	-.04091	-.00667	.00149	.00209
7.5	0.00000	-.11683	-.04244	-.01002	-.00023	.00154
7.6	.30902	-.07488	-.03983	-.01229	-.00184	.00088
7.7	.58779	-.02620	-.03344	-.01328	-.00318	.00018
7.8	.80902	.02440	-.02399	-.01296	-.00416	-.00050
7.9	.95106	.07198	-.01245	-.01141	-.00469	-.00108
8.0	1.00000	.11197	0.00000	-.00882	-.00475	-.00153
8.1	.95106	.14056	.01214	-.00548	-.00435	-.00180
8.2	.80902	.15513	.02282	-.00173	-.00355	-.00189
8.3	.58779	.15443	.03103	.00205	-.00246	-.00179
8.4	.30902	.13871	.03604	.00549	-.00118	-.00153
8.5	0.00000	.10968	.03745	.00829	.00016	-.00114
8.6	-.30902	.07031	.03520	.01020	.00141	-.00066
8.7	-.58779	.02456	.02960	.01106	.00248	-.00014
8.8	-.80902	-.02306	.02126	.01082	.00326	.00035
8.9	-.95106	-.06790	.01105	.00955	.00369	.00079
9.0	-1.00000	-.10563	0.00000	.00741	.00375	.00113
9.1	-.95106	-.13265	-.01081	.00462	.00345	.00134
9.2	-.80902	-.14647	-.02034	.00148	.00283	.00142
9.3	-.58779	-.14588	-.02769	-.00171	.00197	.00135
9.4	-.30902	-.13109	-.03221	-.00463	.00095	.00116
9.5	0.00000	-.10370	-.03351	-.00701	-.00011	.00087
9.6	.30902	-.06649	-.03153	-.00864	-.00112	.00051
9.7	.58779	-.02319	-.02655	-.00939	-.00198	.00012
9.8	.80902	.02192	-.01909	-.00921	-.00262	-.00026
9.9	.95106	.06444	-.00996	-.00815	-.00299	-.00060
10.0	1.00000	.10025	0.00000	-.00633	-.00304	-.00086
10.1	.95106	.12595	.00974	-.00396	-.00280	-.00103
10.2	.80902	.13912	.01834	-.00127	-.00231	-.00109
10.3	.58779	.13861	.02500	.00146	-.00161	-.00105
10.4	.30902	.12461	.02911	.00397	-.00079	-.00090
10.5	0.00000	.09860	.03032	.00602	.00008	-.00068
10.6	-.30902	.06323	.02856	.00744	.00091	-.00040
10.7	-.58779	.02203	.02407	.00810	.00162	-.00010
10.8	-.80902	-.02093	.01732	.00796	.00215	.00020
10.9	-.95106	-.06145	.00902	.00706	.00246	.00047
11.0	-1.00000	-.09562	0.00000	.00549	.00251	.00068
11.1	-.95106	-.12016	-.00886	.00344	.00232	.00081
11.2	-.80902	-.13277	-.01671	.00112	.00192	.00087
11.3	-.58779	-.13233	-.02279	-.00126	.00134	.00083
11.4	-.30902	-.11900	-.02656	-.00345	.00066	.00072
11.5	0.00000	-.09418	-.02768	-.00525	-.00006	.00055
11.6	.30902	-.06040	-.02610	-.00650	-.00076	.00032
11.7	.58779	-.02103	-.02201	-.00708	-.00135	.00008
11.8	.80902	.02006	-.01586	-.00697	-.00180	-.00016
11.9	.95106	.05885	-.00827	-.00619	-.00206	-.00037
12.0	1.00000	.09158	0.00000	-.00483	-.00211	-.00054

Table 3. Lambda-Struve functions

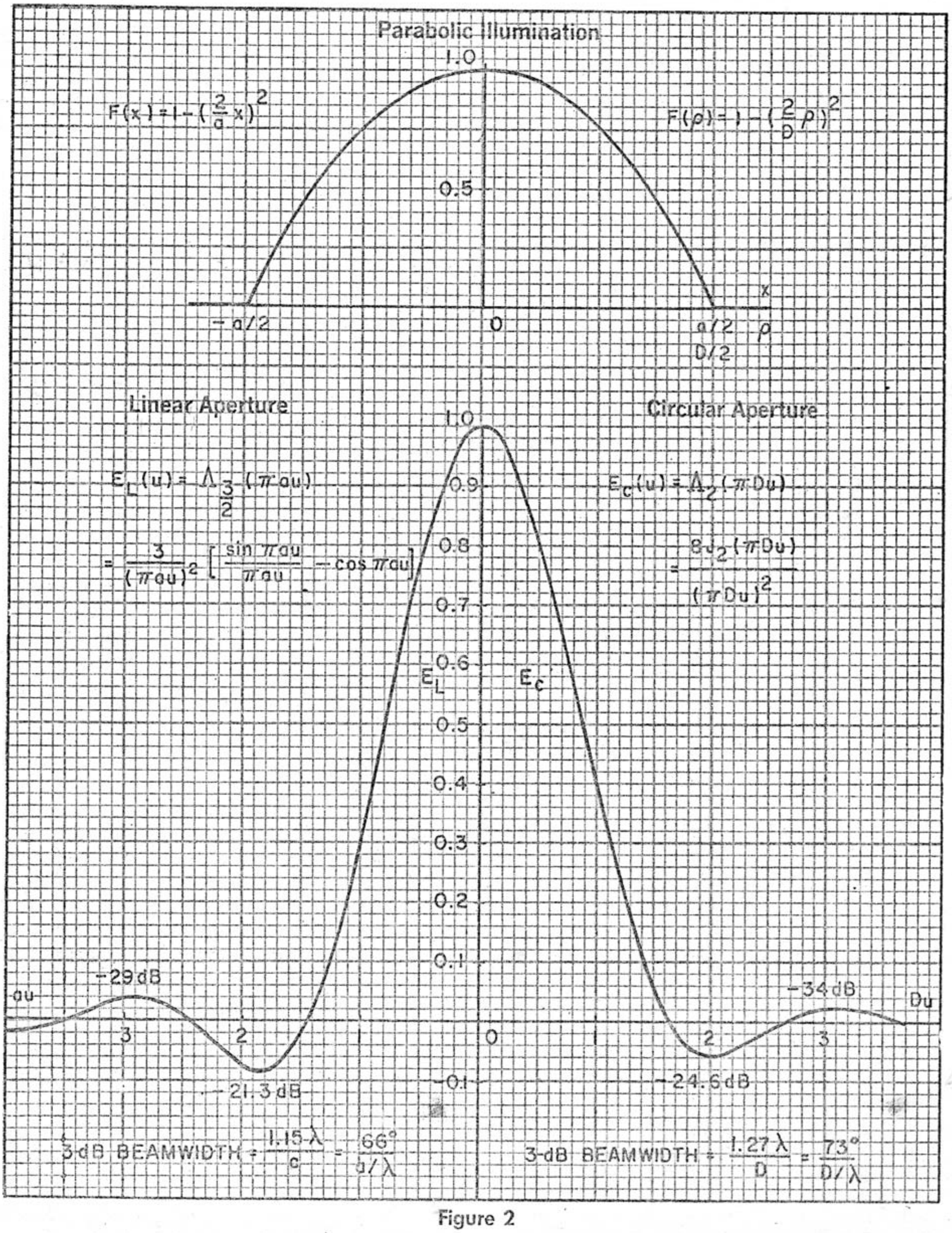
X	$\Lambda H_{-\frac{1}{2}}(\pi X)$	$\Lambda H_0(\pi X)$	$\Lambda H_{\frac{1}{2}}(\pi X)$	$\Lambda H_1(\pi X)$	$\Lambda H_{\frac{3}{2}}(\pi X)$	$\Lambda_2(\pi X)$
	$\sin \pi X$	$H_0(\pi X)$	$\frac{1-\cos \pi X}{\pi X}$	$\frac{2H_1(\pi X)}{\pi X}$	SEE TABLE 1	$\frac{8H_2(\pi X)}{(\pi X)^2}$
0.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
.1	.30902	.19782	.15579	.13246	.11717	.10617
.2	.58779	.38273	.30396	.25973	.23050	.20936
.3	.80902	.54285	.43737	.37691	.33637	.30673
.4	.95106	.66822	.54987	.47966	.43149	.39569
.5	1.00000	.75158	.63662	.56444	.51311	.47400
.6	.95106	.78891	.69446	.62869	.57912	.53988
.7	.80902	.77970	.72201	.67096	.62812	.59208
.8	.58779	.72692	.71979	.69092	.65952	.62989
.9	.30902	.63670	.69005	.68941	.67350	.65317
1.0	0.00000	.51783	.63662	.66828	.67097	.66233
1.1	-.30902	.38090	.56458	.63027	.65352	.65831
1.2	-.58779	.23748	.47906	.57883	.62325	.64246
1.3	-.80902	.09910	.38878	.51783	.58272	.61651
1.4	-.95106	-.02365	.29742	.45136	.53470	.58243
1.5	-1.00000	-.12200	.21221	.38342	.48207	.54233
1.6	-.95106	-.18964	.13747	.31775	.42766	.49839
1.7	-.80902	-.22314	.07718	.25755	.37407	.45269
1.8	-.58779	-.22216	.03377	.20537	.32357	.40717
1.9	-.30902	-.18934	.00820	.16300	.27801	.36352
2.0	0.00000	-.12996	0.00000	.13137	.23873	.32315
2.1	.30902	-.05139	.00742	.11064	.20658	.28711
2.2	.58779	.03767	.02763	.10019	.18185	.25610
2.3	.80902	.12798	.05705	.09877	.16439	.23047
2.4	.95106	.21063	.09164	.10463	.15359	.21024
2.5	1.00000	.27781	.12732	.11571	.14854	.19513
2.6	.95106	.32346	.16026	.12982	.14808	.18462
2.7	.80902	.34381	.18719	.14481	.15091	.17801
2.8	.58779	.33759	.20565	.15870	.15571	.17450
2.9	.30902	.30608	.21415	.16989	.16121	.17321
3.0	0.00000	.25290	.21221	.17717	.16632	.17331
3.1	-.30902	.18363	.20034	.17983	.17013	.17400
3.2	-.58779	.10516	.17995	.17767	.17200	.17461
3.3	-.80902	.02507	.15315	.17093	.17154	.17461
3.4	-.95106	-.04912	.12255	.16027	.16866	.17362
3.5	-1.00000	-.11067	.09095	.14666	.16349	.17142
3.6	-.95106	-.15422	.06110	.13127	.15637	.16796
3.7	-.80902	-.17625	.03546	.11535	.14779	.16331
3.8	-.58779	-.17538	.01600	.10013	.13836	.15767
3.9	-.30902	-.15243	.00399	.08668	.12868	.15130
4.0	0.00000	-.11031	0.00000	.07585	.11937	.14453
4.1	.30902	-.05366	.00380	.06820	.11094	.13768
4.2	.58779	.01159	.01447	.06399	.10380	.13107
4.3	.80902	.07885	.03051	.06314	.09824	.12498
4.4	.95106	.14149	.04999	.06528	.09437	.11962
4.5	1.00000	.19349	.07074	.06983	.09215	.11513
4.6	.95106	.22998	.09058	.07600	.09144	.11158
4.7	.80902	.24767	.10753	.08292	.09194	.10893
4.8	.58779	.24518	.11996	.08973	.09330	.10711
4.9	.30902	.22308	.12674	.09563	.09513	.10598
5.0	0.00000	.18381	.12732	.09995	.09704	.10536
5.1	-.30902	.13144	.12177	.10222	.09865	.10505
5.2	-.58779	.07119	.11074	.10220	.09967	.10485
5.3	-.80902	.00897	.09536	.09987	.09987	.10458
5.4	-.95106	-.04926	.07716	.09544	.09914	.10409
5.5	-1.00000	-.09799	.05767	.08931	.09744	.10327
5.6	-.95106	-.13277	.03928	.08202	.09486	.10205
5.7	-.80902	-.15054	.02302	.07418	.09155	.10041
5.8	-.58779	-.14994	.01048	.06646	.08773	.09838
5.9	-.30902	-.13141	.00264	.05945	.08365	.09603
6.0	0.00000	-.09710	0.00000	.05367	.07958	.09346
6.1	.30902	-.05066	.00255	.04950	.07577	.09077
6.2	.58779	.00318	.00981	.04714	.07244	.08809
6.3	.80902	.05902	.02083	.04664	.06975	.08552
6.4	.95106	.11137	.03437	.04786	.06780	.08318
6.5	1.00000	.15517	.04897	.05049	.06661	.08113
6.6	.95106	.18625	.06313	.05415	.06615	.07942
6.7	.80902	.20176	.07543	.05833	.06630	.07806
6.8	.58779	.20037	.08468	.06254	.06691	.07705
6.9	.30902	.18243	.09001	.06628	.06780	.07634
7.0	0.00000	.14987	.09095	.06915	.06877	.07586
7.1	-.30902	.10601	.08747	.07081	.06964	.07554
7.2	-.58779	.05525	.07998	.07109	.07023	.07530
7.3	-.80902	.00254	.06923	.06994	.07042	.07505
7.4	-.95106	-.04699	.05631	.06745	.07011	.07472
7.5	-1.00000	-.08863	.04244	.06385	.06930	.07425
7.6	-.95106	-.11847	.02894	.05944	.06798	.07360
7.7	-.80902	-.13381	.01704	.05462	.06624	.07276
7.8	-.58779	-.13337	.00779	.04979	.06419	.07172
7.9	-.30902	-.11744	.00197	.04535	.06195	.07052
8.0	0.00000	-.08780	0.00000	.04165	.05968	.06919
8.1	.30902	-.04751	.00192	.03895	.05752	.06778
8.2	.58779	-.00067	.00741	.03741	.05560	.06636
8.3	.80902	.04808	.01581	.03709	.05403	.06498
8.4	.95106	.09394	.02618	.03790	.05286	.06369
8.5	1.00000	.13247	.03745	.03968	.05212	.06253
8.6	.95106	.15997	.04865	.04217	.05181	.06154
8.7	.80902	.17388	.05809	.04505	.05187	.06073
8.8	.58779	.17297	.06543	.04798	.05221	.06010
8.9	.30902	.15748	.06978	.05063	.05273	.05962
9.0	0.00000	.12905	.07074	.05269	.05332	.05928
9.1	-.30902	.09056	.06825	.05396	.05385	.05902
9.2	-.58779	.04585	.06259	.05427	.05423	.05882
9.3	-.80902	-.00070	.05434	.05359	.05437	.05861
9.4	-.95106	-.04457	.04433	.05197	.05422	.05837
9.5	-1.00000	-.08153	.03351	.04954	.05374	.05806
9.6	-.95106	-.10808	.02291	.04653	.05295	.05765
9.7	-.80902	-.12179	.01353	.04320	.05188	.05713
9.8	-.58779	-.12145	.00620	.03983	.05060	.05650
9.9	-.30902	-.10728	.00157	.03670	.04919	.05577
10.0	0.00000	-.08081	0.00000	.03408	.04775	.05497
10.1	.30902	-.04475	.00154	.03216	.04636	.05411
10.2	.58779	-.00274	.00596	.03107	.04511	.05324
10.3	.80902	.04106	.01274	.03084	.04407	.05239
10.4	.95106	.08236	.02115	.03144	.04330	.05158
10.5	1.00000	.11714	.03032	.03275	.04280	.05084
10.6	.95106	.14205	.03931	.03458	.04258	.05020
10.7	.80902	.15476	.04723	.03672	.04260	.04967
10.8	.58779	.15411	.05332	.03891	.04282	.04923
10.9	.30902	.14028	.05698	.04091	.04316	.04890
11.0	0.00000	.11472	.05787	.04249	.04355	.04864
11.1	-.30902	.08001	.05595	.04349	.04392	.04844
11.2	-.58779	.03959	.05141	.04378	.04418	.04828
11.3	-.80902	-.00256	.04473	.04333	.04429	.04811
11.4	-.95106	-.04235	.03655	.04218	.04419	.04793
11.5	-1.00000	-.07593	.02768	.04041	.04388	.04770
11.6	-.95106	-.10010	.01896	.03819	.04335	.04742
11.7	-.80902	-.11260	.01121	.03571	.04263	.04706
11.8	-.58779	-.11233	.00515	.03319	.04176	.04664
11.9	-.30902	-.09945	.00131	.03085	.04079	.04616
12.0	0.00000	-.07531	0.00000	.02887	.03979	.04562



Uniform Illumination
1.0
F(x) = 1
x
−a/2
0
a/2
D/2
ρ
Linear Aperture
Circular Aperture
$E_L(u) = \Lambda_{\frac{1}{2}}(\pi a u)$
$= \frac{\sin \pi a u}{\pi a u}$
$u = \frac{\sin\theta}{\lambda}$
$E_c(u) = \Lambda_1(\pi D u)$
$= \frac{2 J_1(\pi D u)}{\pi D u}$
$u = \frac{\sin\theta}{\lambda}$
1.0
0.9
0.8
0.7
0.6
E_L
E_c
0.5
0.4
0.3
0.2
0.1
−0.1
−0.2
−17.8 dB
−24 dB
au
Du
3
2
1
0
2
3
−28 dB
−20.8 dB
−17.6 dB
−13.3 dB
3-dB BEAMWIDTH = $\frac{0.89\lambda}{a} = \frac{51°}{a/\lambda}$
3-dB BEAMWIDTH = $\frac{1.02\lambda}{D} = \frac{58.4°}{D/\lambda}$

Figure 1

V



Parabolic Illumination
1.0
0.5
$F(x) = 1 - (\frac{2}{a}x)^2$
$F(\rho) = 1 - (\frac{2}{D}\rho)^2$
−a/2
0
a/2
D/2
x
ρ
Linear Aperture
Circular Aperture
$E_L(u) = \Lambda_{\frac{3}{2}}(\pi a u)$
$= \frac{3}{(\pi a u)^2}\left[\frac{\sin \pi a u}{\pi a u} - \cos \pi a u\right]$
$E_c(u) = \Lambda_2(\pi D u)$
$= \frac{8 J_2(\pi D u)}{(\pi D u)^2}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
E_L
E_c
−29 dB
−21.3 dB
−34 dB
−24.6 dB
au
Du
3
2
2
3
3-dB BEAMWIDTH = $\frac{1.15\lambda}{a} = \frac{66°}{a/\lambda}$
3-dB BEAMWIDTH = $\frac{1.27\lambda}{D} = \frac{73°}{D/\lambda}$

Figure 2

V

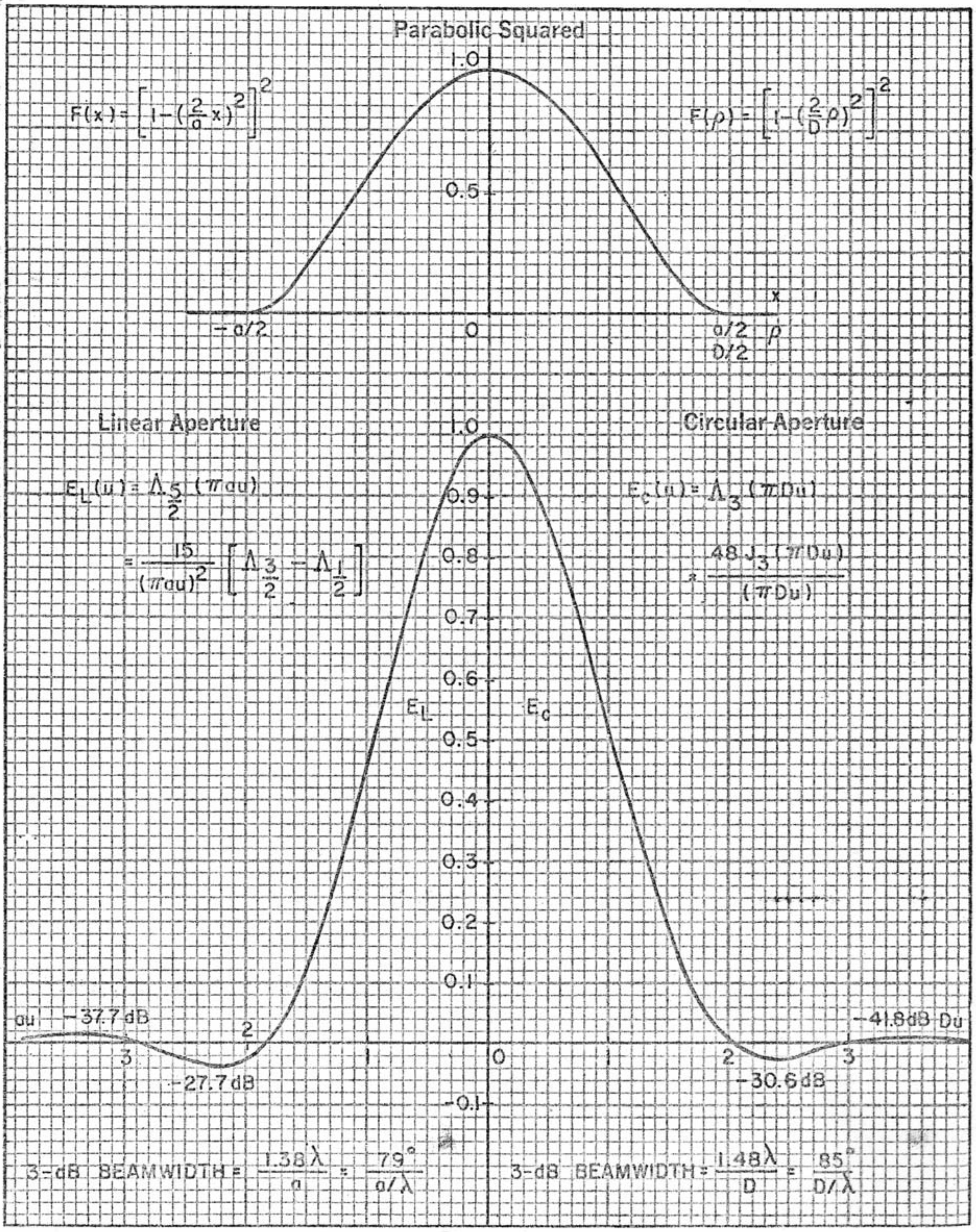
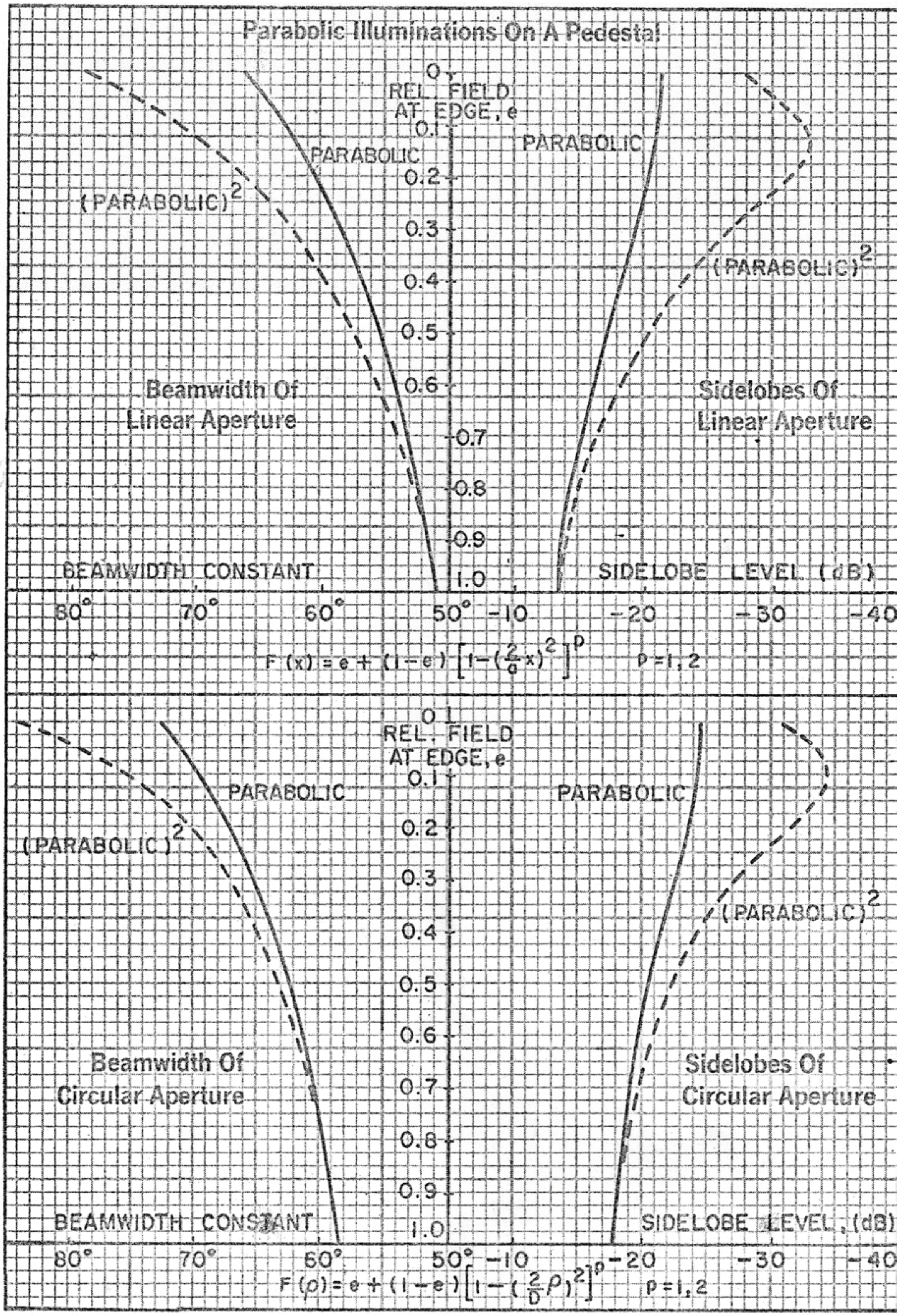


Figure 3

$\frac{15}{(\pi a u)^2}\left[\frac{\sin \pi a u}{\pi a u} - 2\cos \pi a u\right]$



Parabolic Illuminations On A Pedestal
REL. FIELD AT EDGE, e
PARABOLIC
(PARABOLIC)^2
Beamwidth Of Linear Aperture
Sidelobes Of Linear Aperture
BEAMWIDTH CONSTANT
SIDELOBE LEVEL (dB)
$F(x) = e + (1-e)\left[1-\left(\frac{2}{a}x\right)^2\right]^p \quad p=1,2$
Beamwidth Of Circular Aperture
Sidelobes Of Circular Aperture
$F(\rho) = e + (1-e)\left[1-\left(\frac{2}{D}\rho\right)^2\right]^p \quad p=1,2$

Figure 4

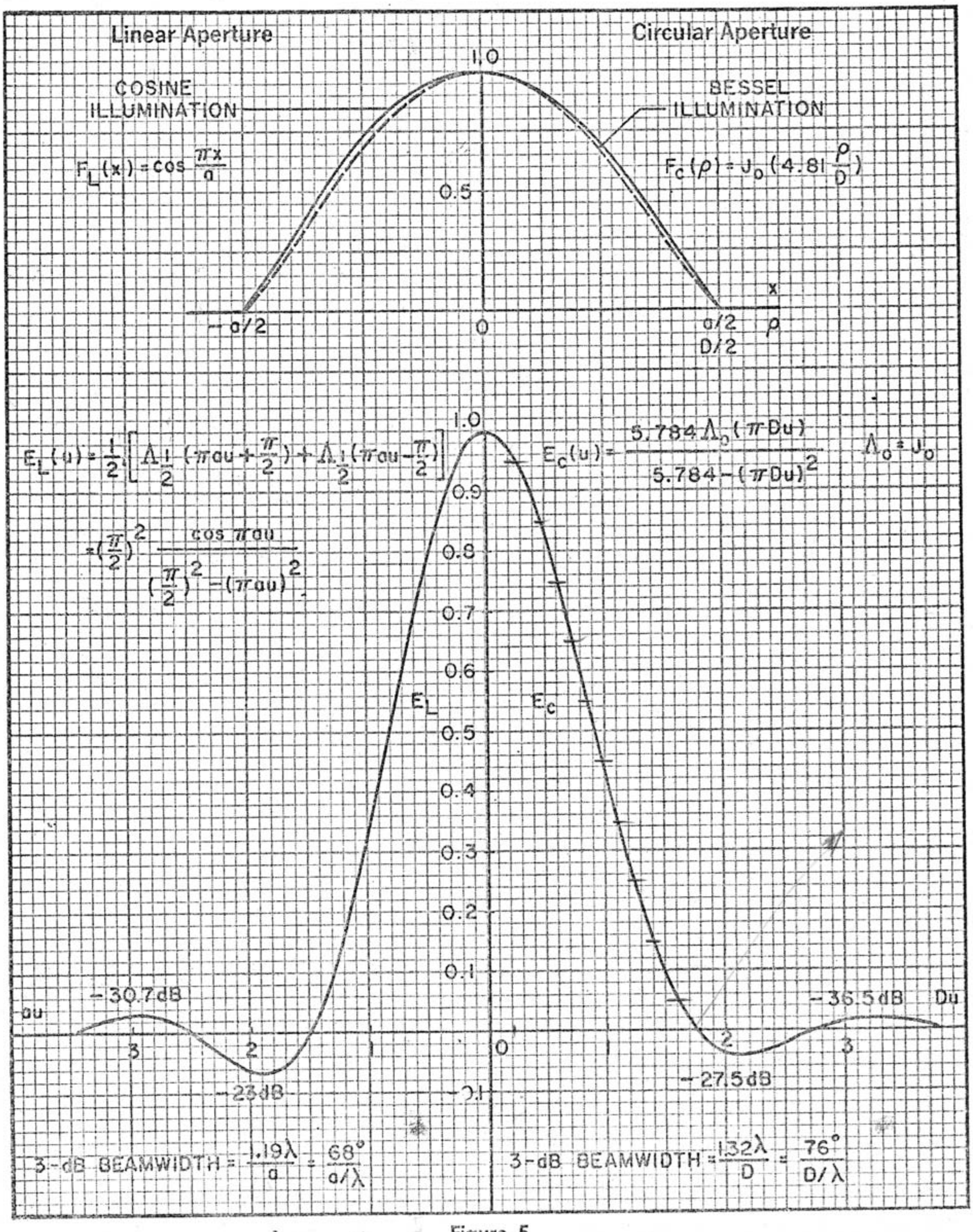
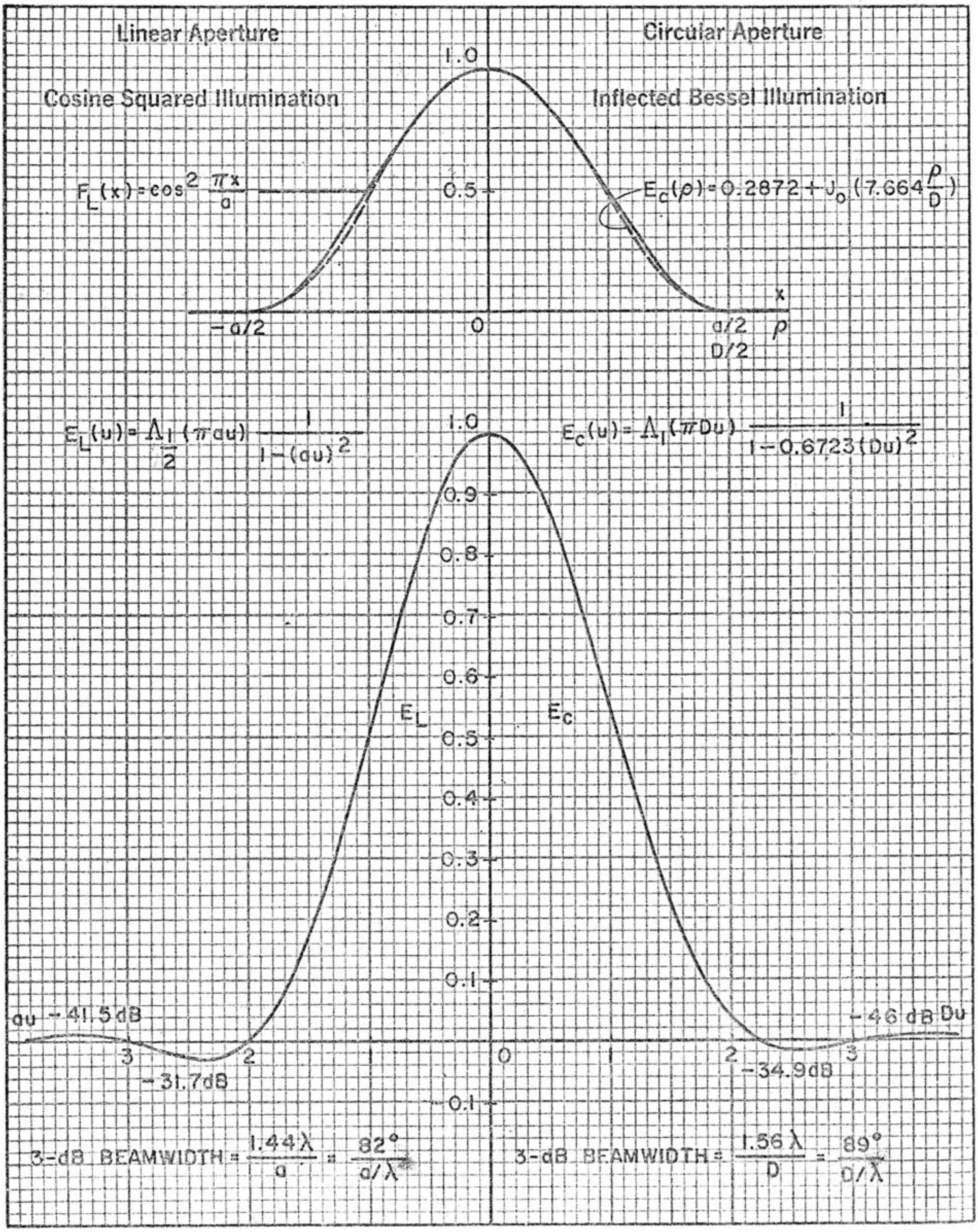


Figure 5

$u = \frac{\text{sen}\,\theta}{\lambda}$

√



Linear Aperture
Cosine Squared Illumination
Circular Aperture
Inflected Bessel Illumination
$F_L(x) = \cos^2 \frac{\pi x}{a}$
$E_c(\rho) = 0.2872 + J_0 (7.664 \frac{\rho}{D})$
1.0
0.5
$-a/2$
0
$a/2$
$D/2$
x
ρ
$E_L(u) = \frac{\Lambda_1}{2} (\pi a u) \frac{1}{1-(au)^2}$
$E_c(u) = \Lambda_1 (\pi D u) \frac{1}{1-0.6723 (Du)^2}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-0.1
E_L
E_c
au
−41.5dB
−31.7dB
3
2
0
1
2
3
−34.9dB
−46 dB
Du
3-dB BEAMWIDTH = $\frac{1.44\lambda}{a} = \frac{82°}{a/\lambda}$
3-dB BEAMWIDTH = $\frac{1.56\lambda}{D} = \frac{89°}{D/\lambda}$

Figure 6

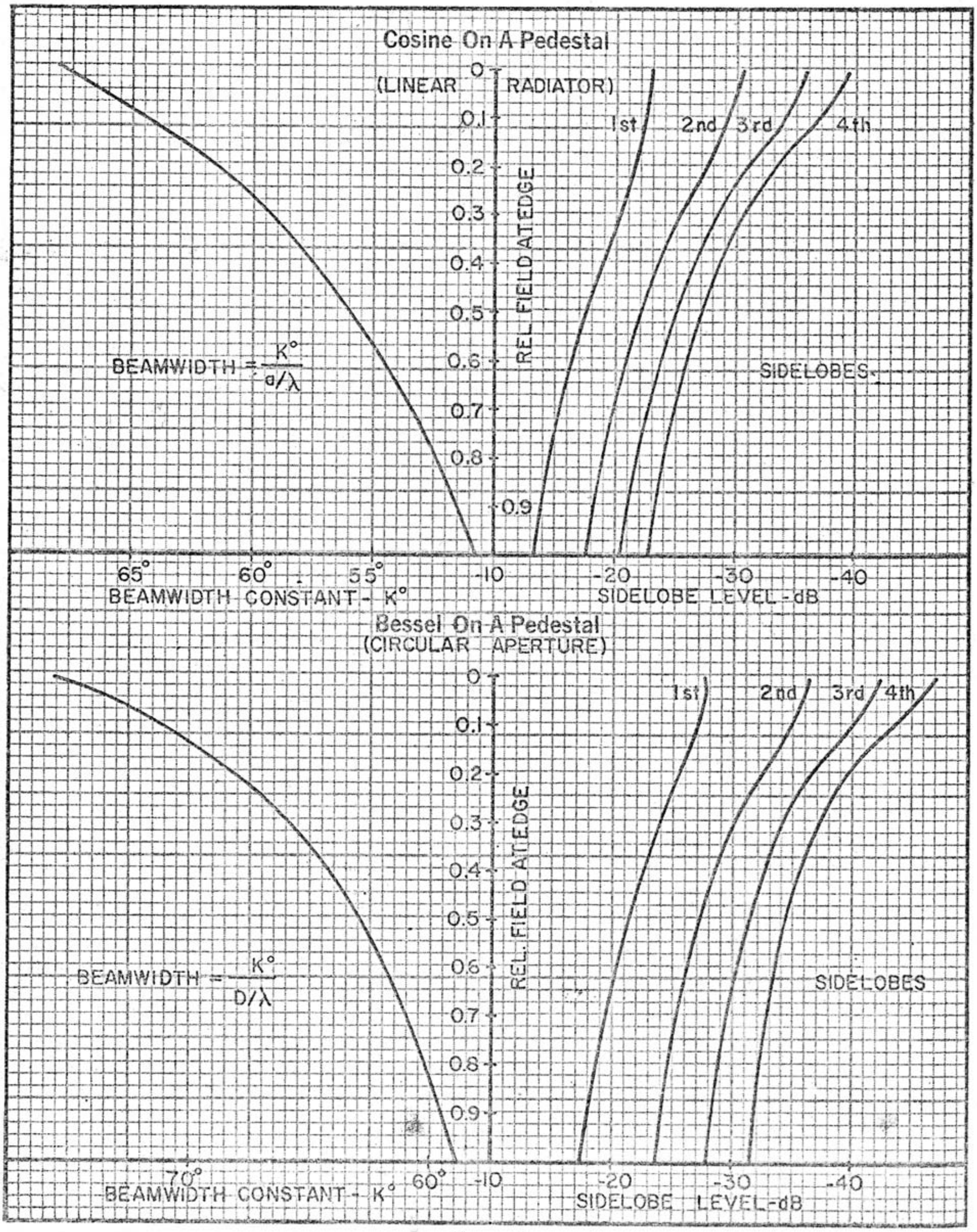

Cosine On A Pedestal
(LINEAR RADIATOR)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
REL. FIELD AT EDGE
1st
2nd
3rd
4th
BEAMWIDTH = K°/(a/λ)
SIDELOBES
65°
60°
55°
-10
-20
-30
-40
BEAMWIDTH CONSTANT - K°
SIDELOBE LEVEL - dB
Bessel On A Pedestal
(CIRCULAR APERTURE)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
REL. FIELD AT EDGE
1st
2nd
3rd
4th
BEAMWIDTH = K°/(D/λ)
SIDELOBES
70°
60°
-10
-20
-30
-40
BEAMWIDTH CONSTANT - K°
SIDELOBE LEVEL - dB

Figure 7

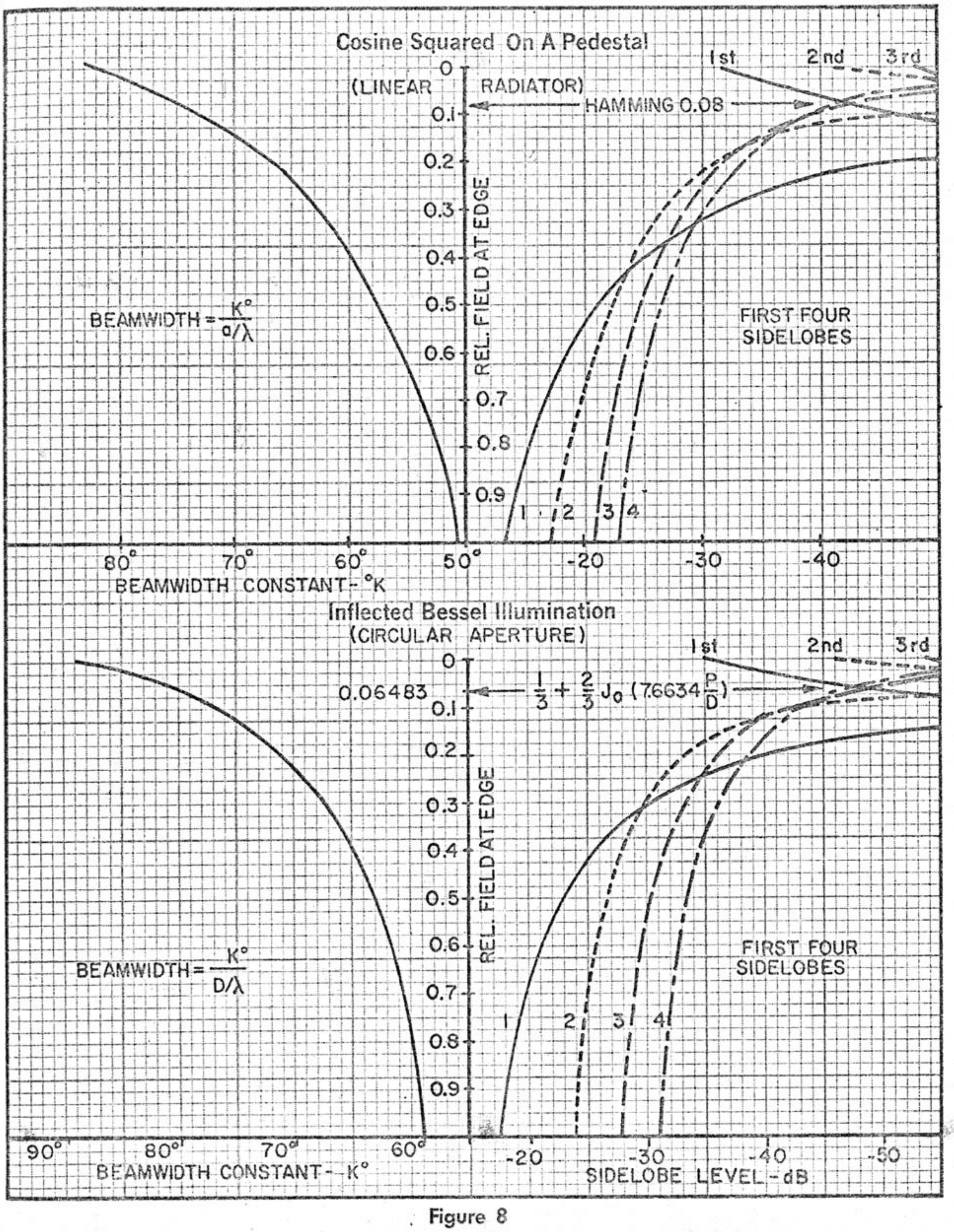

Cosine Squared On A Pedestal
(LINEAR RADIATOR)
HAMMING 0.08
REL. FIELD AT EDGE
BEAMWIDTH = K°/(a/λ)
FIRST FOUR SIDELOBES
1st
2nd
3rd
BEAMWIDTH CONSTANT - °K
Inflected Bessel Illumination
(CIRCULAR APERTURE)
0.06483
1/3 + 2/3 J0 (7.6634 P/D)
BEAMWIDTH = K°/(D/λ)
SIDELOBE LEVEL - dB

Figure 8

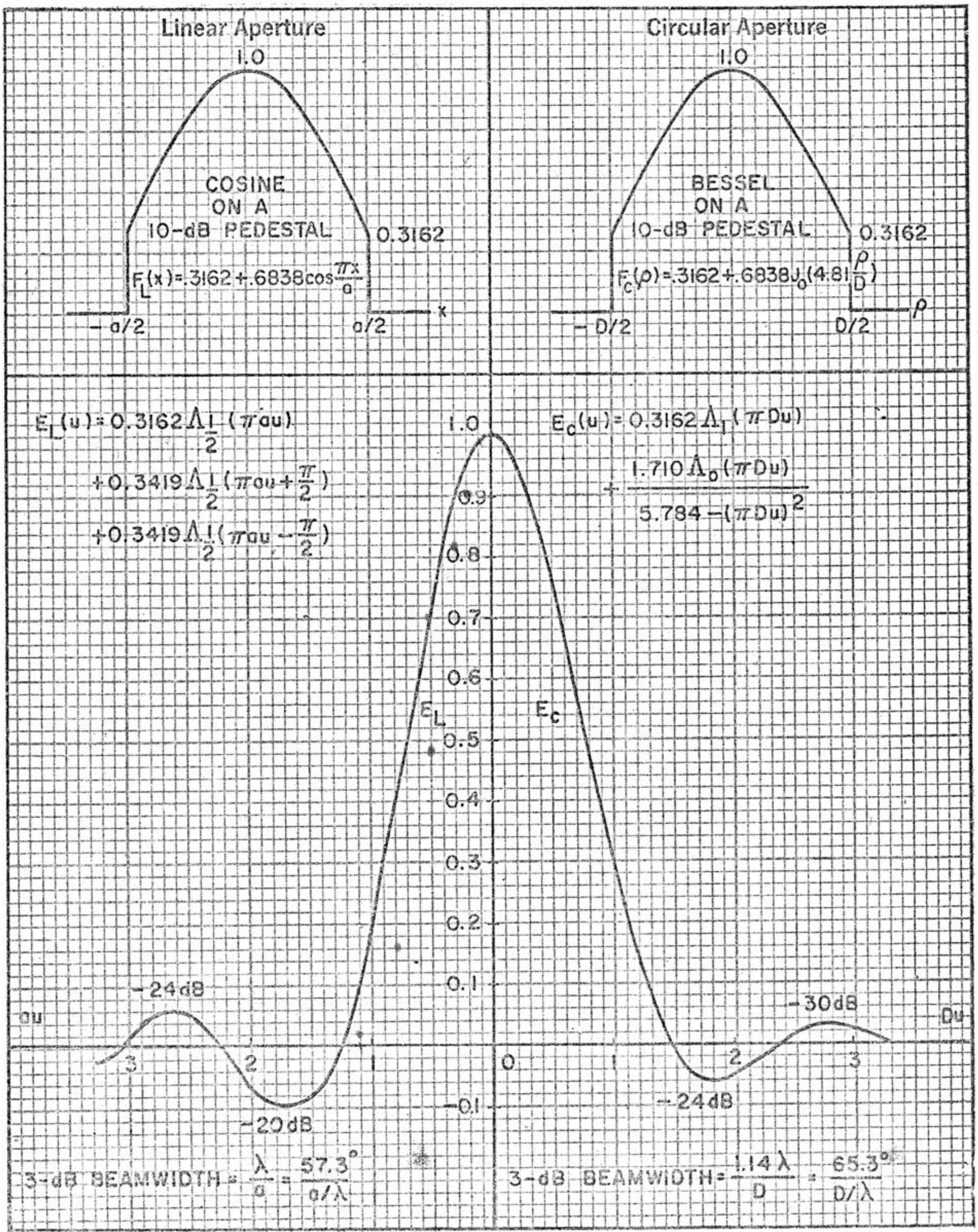
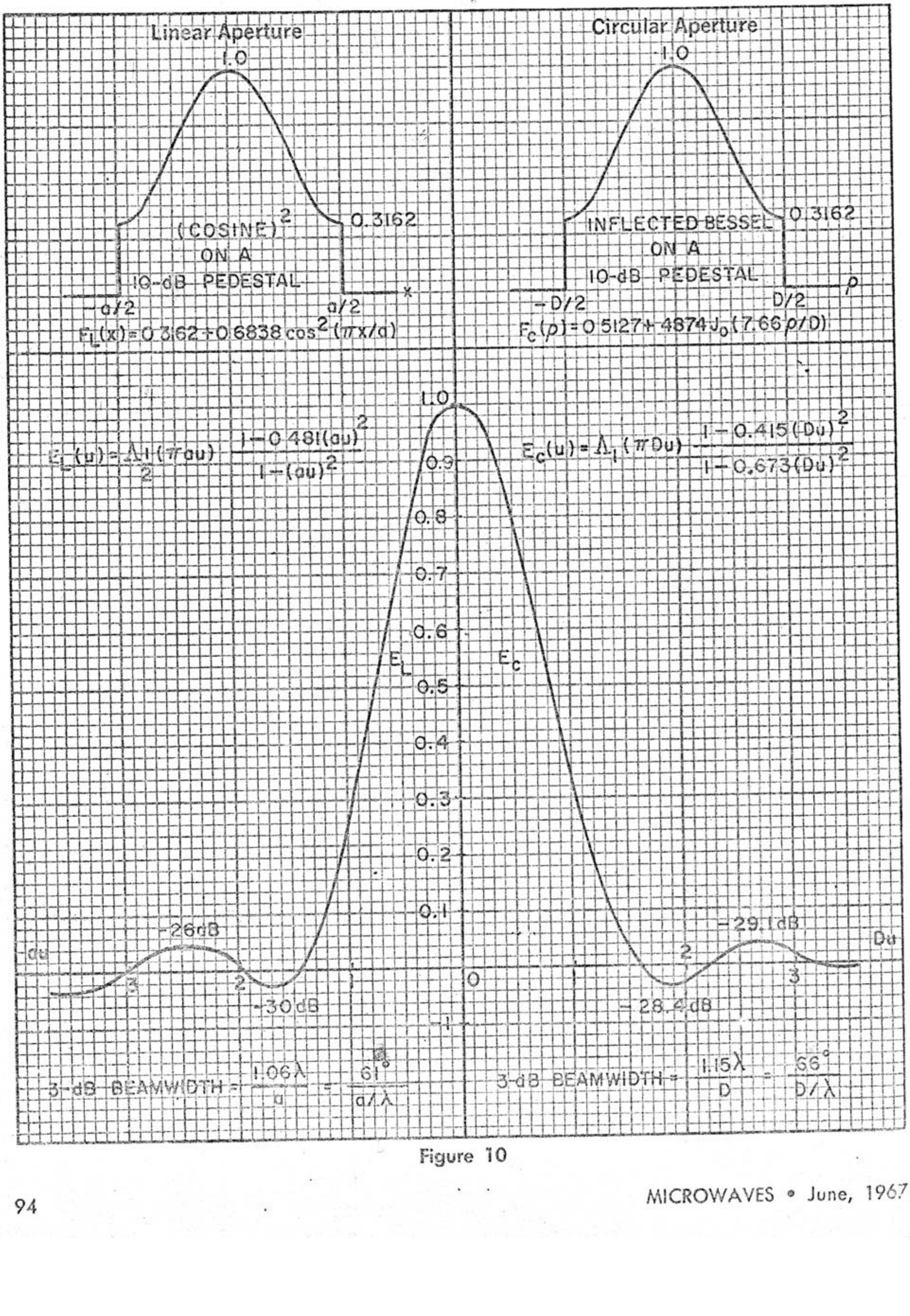


Figure 9

$u = \frac{sen\theta}{\lambda}$



Linear Aperture
1.0
(COSINE)^2
ON A
10-dB PEDESTAL
0.3162
−a/2
a/2
x
$E_L(x) = 0.3162 + 0.6838\cos^2(\pi x/a)$
Circular Aperture
1.0
INFLECTED BESSEL
ON A
10-dB PEDESTAL
0.3162
−D/2
D/2
ρ
$F_c(\rho) = 0.5127 + .4874 J_0(7.66\rho/D)$
$E_L(u) = \frac{\Lambda_1(\pi a u)}{2} \frac{1-0.481(au)^2}{1-(au)^2}$
$E_c(u) = \Lambda_1(\pi D u) \frac{1-0.415(Du)^2}{1-0.673(Du)^2}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
−.1
E_L
E_c
−26dB
−30dB
−29.1dB
−28.4dB
au
Du
3
2
0
2
3
3-dB BEAMWIDTH = $\frac{1.06\lambda}{a} = \frac{61°}{a/\lambda}$
3-dB BEAMWIDTH = $\frac{1.15\lambda}{D} = \frac{66°}{D/\lambda}$

Figure 10

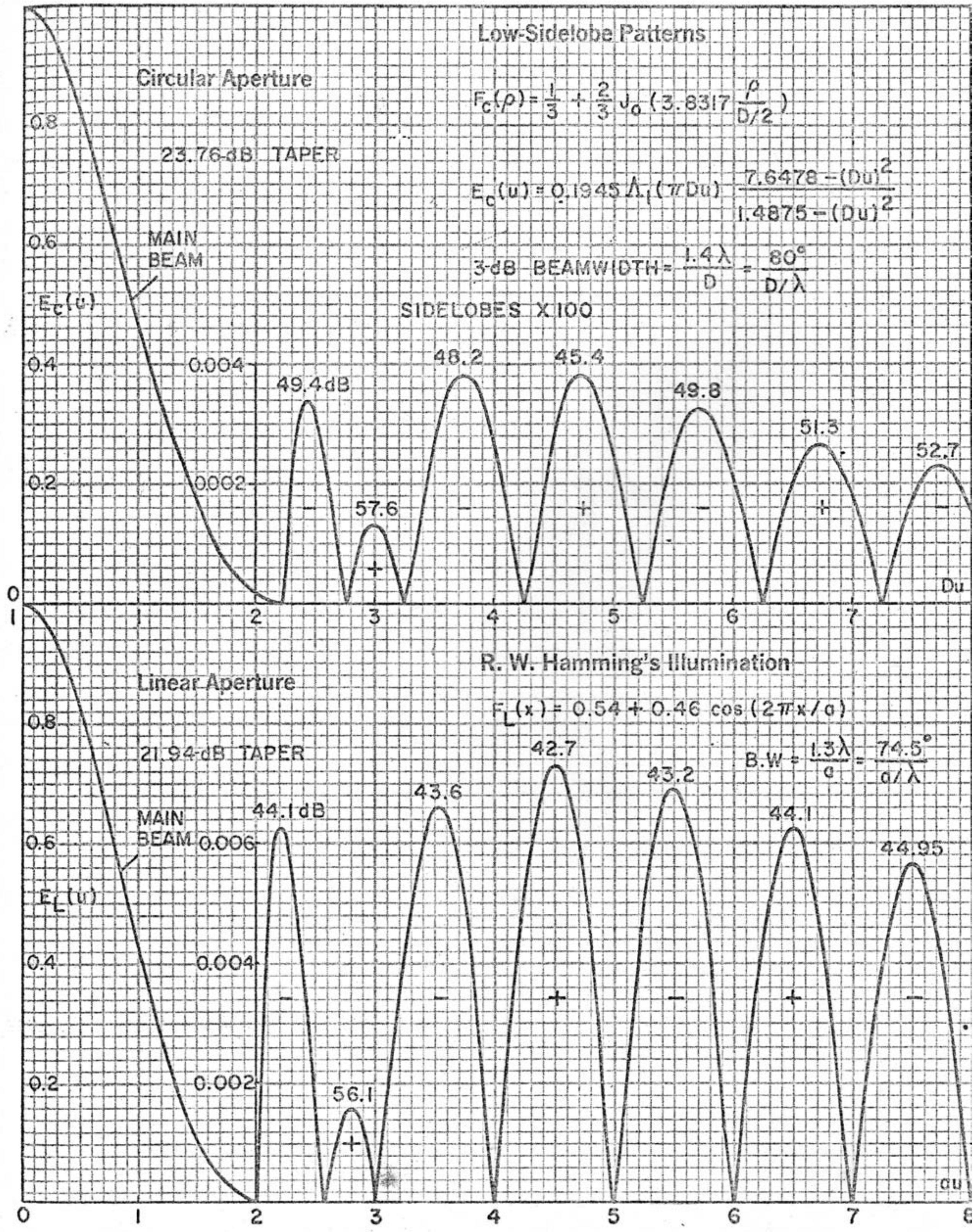
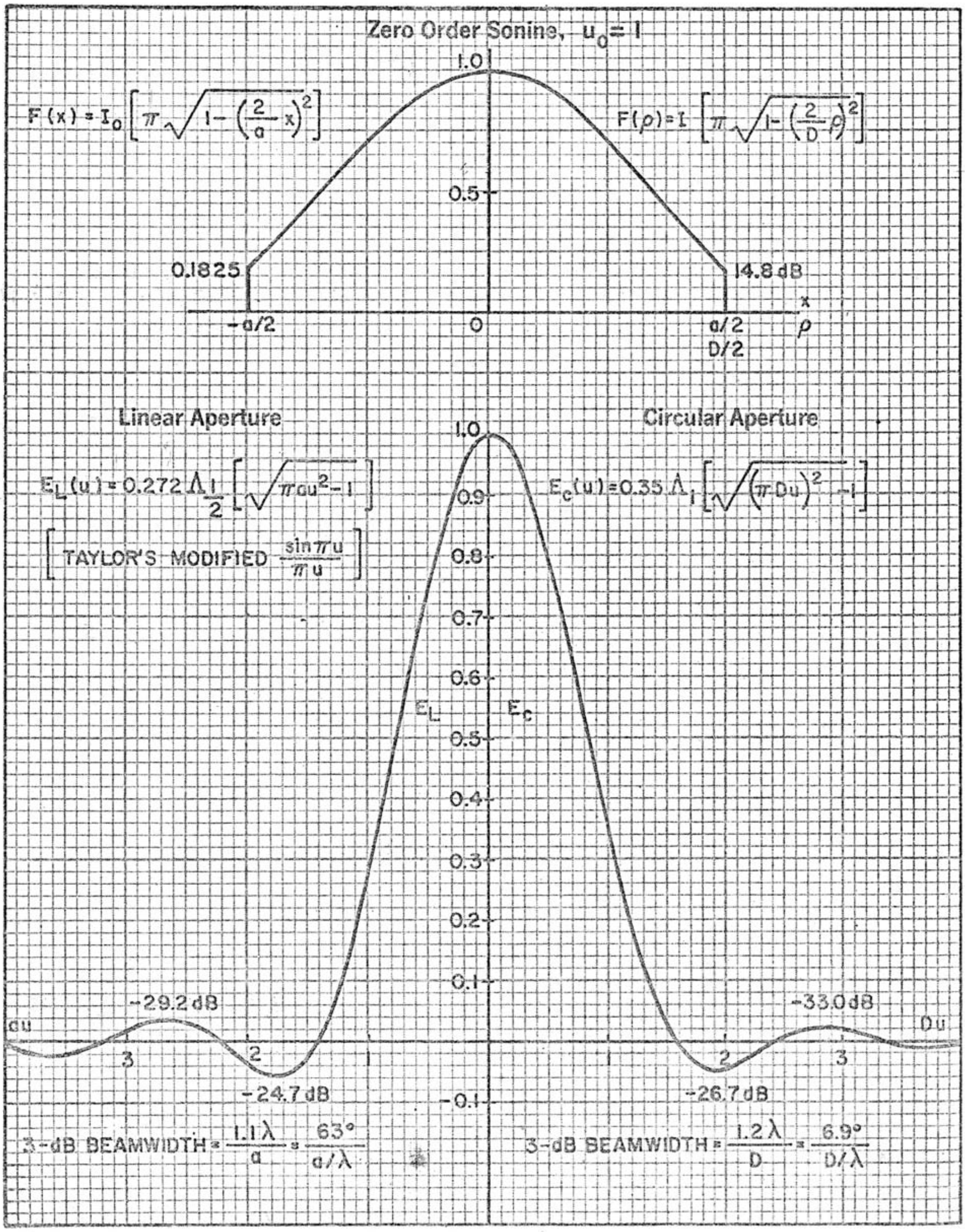

Low-Sidelobe Patterns
Circular Aperture
$F_c(\rho) = \frac{1}{3} + \frac{2}{3} J_0 (3.8317 \frac{\rho}{D/2})$
23.76-dB TAPER
$E_c(u) = 0.1945 \Lambda_1(\pi Du) \frac{7.6478-(Du)^2}{1.4875-(Du)^2}$
MAIN BEAM
3-dB BEAMWIDTH = $\frac{1.4\lambda}{D} = \frac{80°}{D/\lambda}$
SIDELOBES X 100
49.4 dB
48.2
45.4
49.8
51.3
52.7
57.6
Du
R. W. Hamming's Illumination
Linear Aperture
$F_L(x) = 0.54 + 0.46 \cos(2\pi x/a)$
21.94-dB TAPER
B.W = $\frac{1.3\lambda}{a} = \frac{74.5°}{a/\lambda}$
42.7
43.2
43.6
44.1 dB
44.1
44.95
56.1
au

Figure 11

V



Zero Order Sonine, $u_0 = 1$
1.0
0.5
$F(x) = I_0\left[\pi\sqrt{1-\left(\frac{2}{a}x\right)^2}\right]$
$F(\rho) = I\left[\pi\sqrt{1-\left(\frac{2}{D}\rho\right)^2}\right]$
0.1825
14.8 dB
x
ρ
−a/2
0
a/2
D/2
Linear Aperture
Circular Aperture
$E_L(u) = 0.272\,\Lambda_{\frac{1}{2}}\left[\sqrt{\pi a u^2 - 1}\right]$
$E_c(u) = 0.35\,\Lambda_1\left[\sqrt{(\pi D u)^2 - 1}\right]$
$\left[\text{TAYLOR'S MODIFIED } \frac{\sin \pi u}{\pi u}\right]$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
−0.1
E_L
E_c
−29.2 dB
−33.0 dB
au
Du
3
2
1
1
2
3
−24.7 dB
−26.7 dB
3-dB BEAMWIDTH = $\frac{1.1\lambda}{a} = \frac{63°}{a/\lambda}$
3-dB BEAMWIDTH = $\frac{1.2\lambda}{D} = \frac{6.9°}{D/\lambda}$

Figure 12

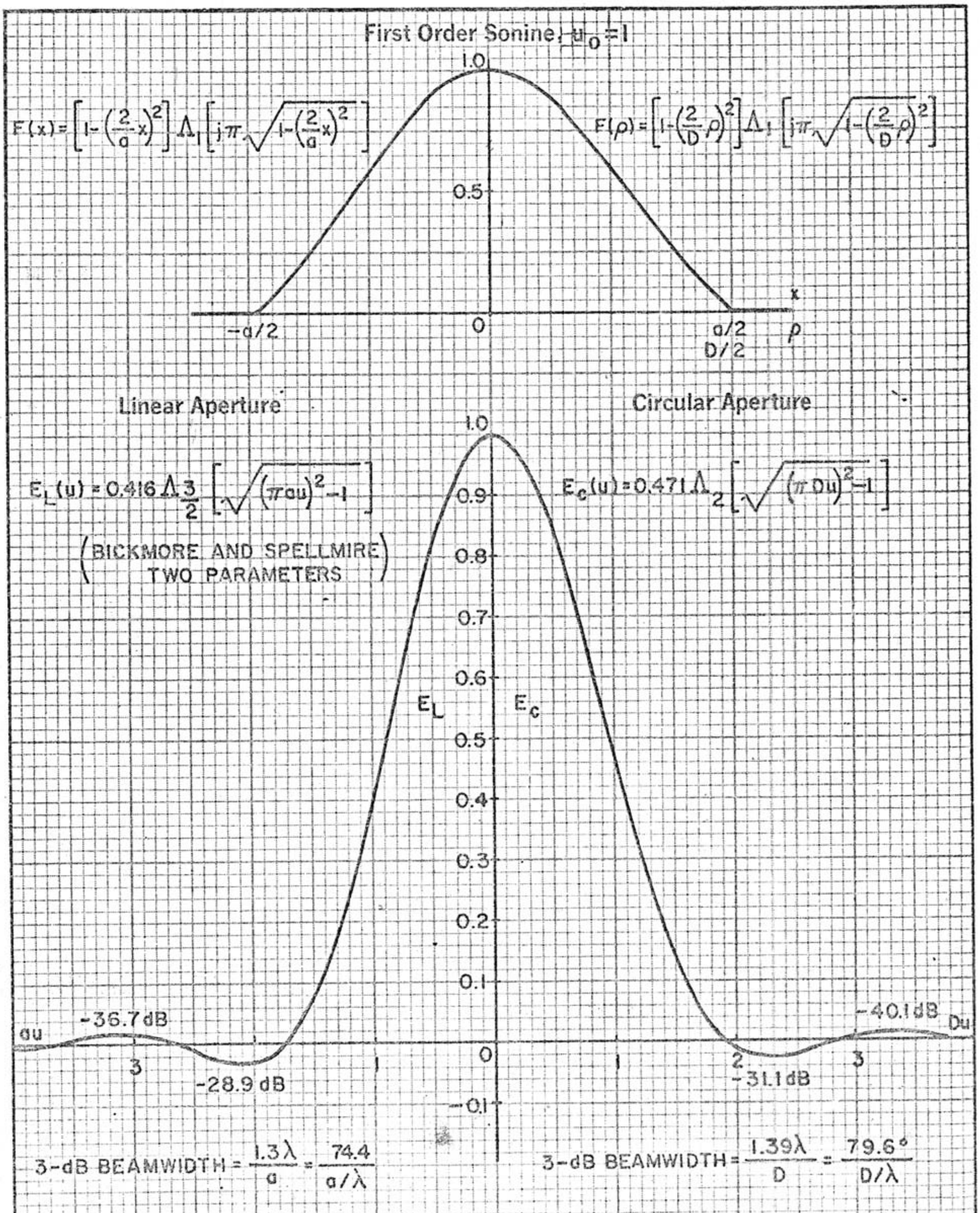

First Order Sonine, $\mu_0 = 1$
$F(x) = \left[1-\left(\frac{2}{a}x\right)^2\right] \Lambda_1 \left[j\pi\sqrt{1-\left(\frac{2}{a}x\right)^2}\right]$
$F(\rho) = \left[1-\left(\frac{2}{D}\rho\right)^2\right] \Lambda_1 \left[j\pi\sqrt{1-\left(\frac{2}{D}\rho\right)^2}\right]$
1.0
0.5
−a/2
0
a/2
D/2
x
ρ
Linear Aperture
Circular Aperture
$E_L(u) = 0.416\,\Lambda_{\frac{3}{2}}\left[\sqrt{(\pi a u)^2 - 1}\right]$
(BICKMORE AND SPELLMIRE TWO PARAMETERS)
$E_c(u) = 0.471\,\Lambda_2\left[\sqrt{(\pi D u)^2 - 1}\right]$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
E_L
E_c
−36.7 dB
−28.9 dB
−31.1 dB
−40.1 dB
au
Du
3
1
2
3-dB BEAMWIDTH = $\frac{1.3\lambda}{a} = \frac{74.4}{a/\lambda}$
3-dB BEAMWIDTH = $\frac{1.39\lambda}{D} = \frac{79.6°}{D/\lambda}$

Figure 13

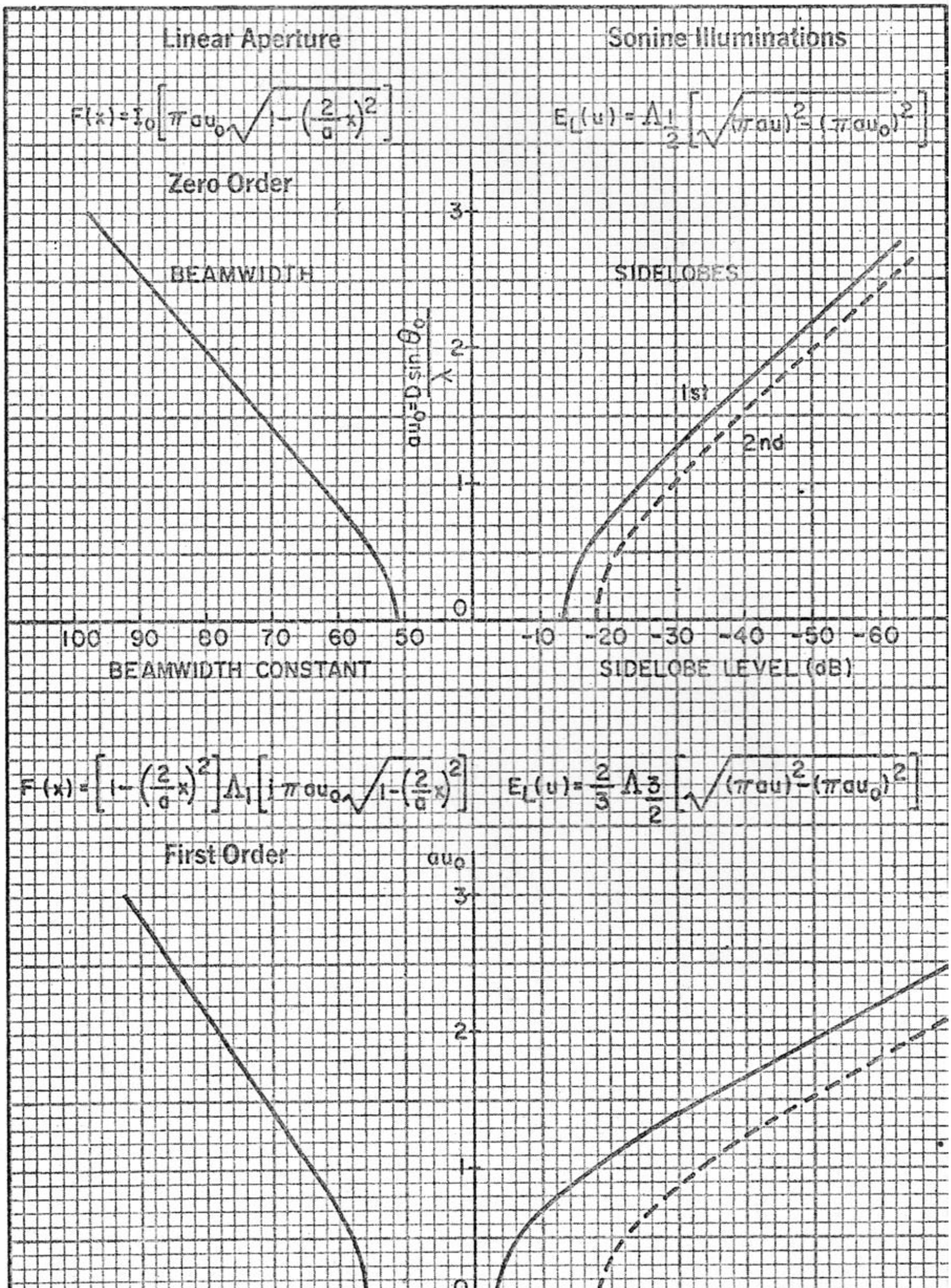

Linear Aperture
Sonine Illuminations
F(x) = I0[π au0 √(1 − (2/a x)²)]
EL(u) = Λ1/2[√((πau)² − (πau0)²)]
Zero Order
BEAMWIDTH
SIDELOBES
au0 = D sin θ0 / λ
1st
2nd
100 90 80 70 60 50
-10 -20 -30 -40 -50 -60
BEAMWIDTH CONSTANT
SIDELOBE LEVEL (dB)
F(x) = [1 − (2/a x)²] Λ1[j π au0 √(1 − (2/a x)²)]
EL(u) = 2/3 Λ3/2[√((πau)² − (πau0)²)]
First Order
au0
110 100 90 80 70 60
-20 -30 -40 -50
BEAMWIDTH CONSTANT
SIDELOBE LEVEL (dB)

Figure 14

V

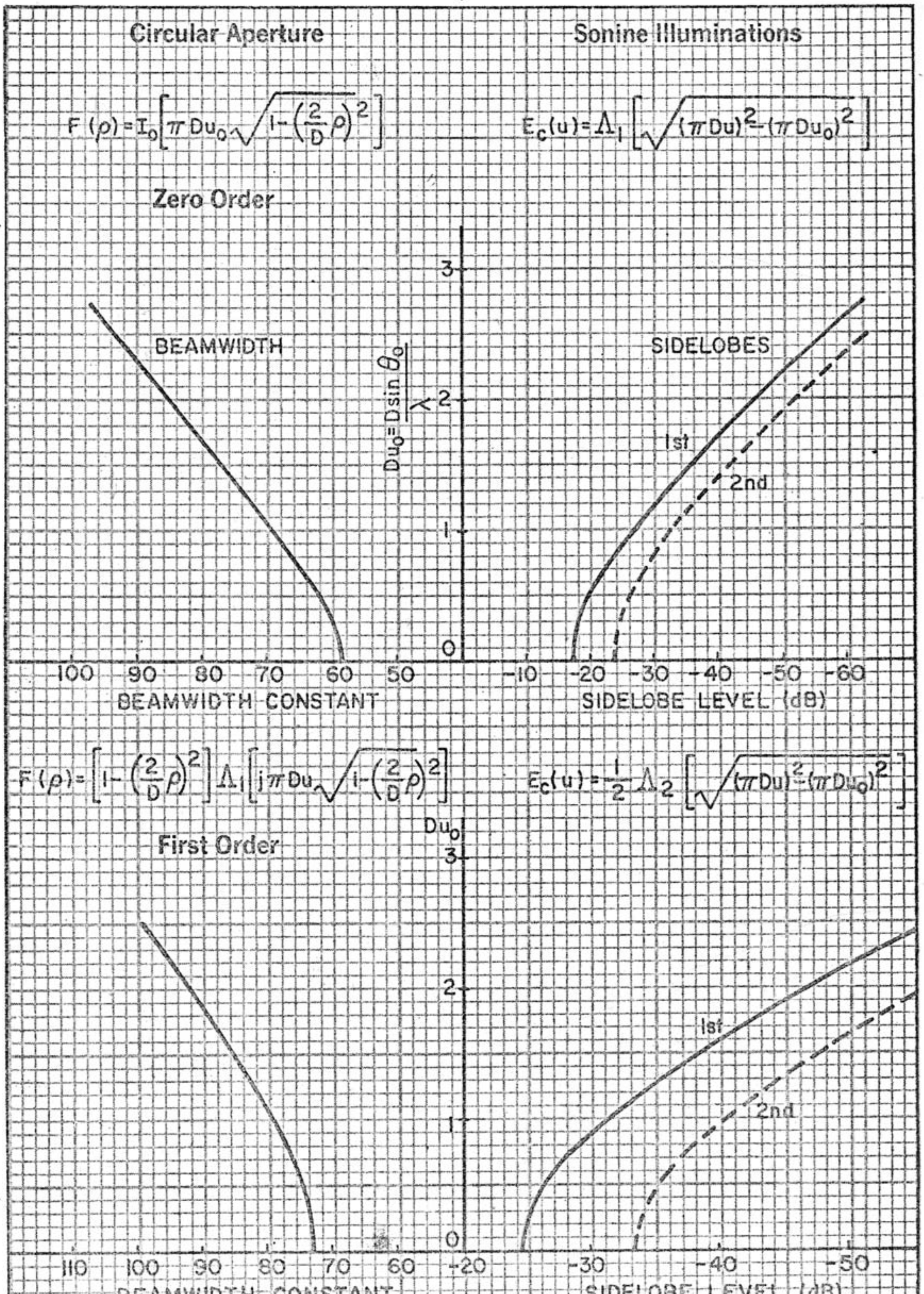
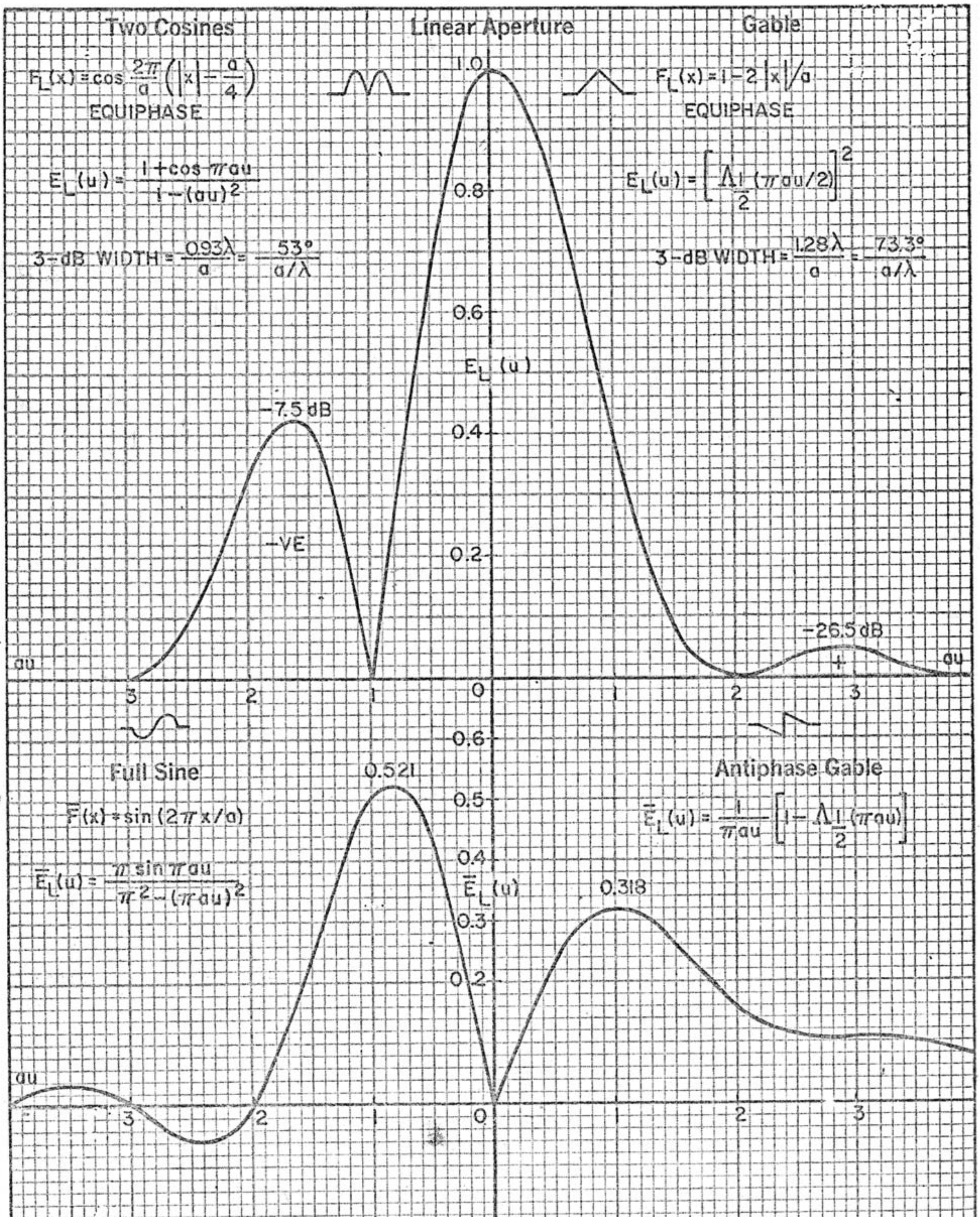

Circular Aperture
Sonine Illuminations
$F(\rho)=I_0\left[\pi Du_0\sqrt{1-\left(\frac{2}{D}\rho\right)^2}\right]$
$E_c(u)=\Lambda_1\left[\sqrt{(\pi Du)^2-(\pi Du_0)^2}\right]$
Zero Order
BEAMWIDTH
SIDELOBES
$Du_0=\frac{D\sin\theta_0}{\lambda}$
1st
2nd
BEAMWIDTH CONSTANT
SIDELOBE LEVEL (dB)
$F(\rho)=\left[1-\left(\frac{2}{D}\rho\right)^2\right]\Lambda_1\left[j\pi Du_0\sqrt{1-\left(\frac{2}{D}\rho\right)^2}\right]$
$E_c(u)=\frac{1}{2}\Lambda_2\left[\sqrt{(\pi Du)^2-(\pi Du_0)^2}\right]$
First Order
Du_0
1st
2nd
BEAMWIDTH CONSTANT
SIDELOBE LEVEL (dB)

Figure 15

√



Two Cosines
$F_L(x) = \cos \frac{2\pi}{a}\left(|x| - \frac{a}{4}\right)$
EQUIPHASE
$E_L(u) = \frac{1 + \cos \pi au}{1 - (au)^2}$
3-dB WIDTH = $\frac{0.93\lambda}{a} = \frac{53°}{a/\lambda}$
Linear Aperture
Gable
$F_L(x) = 1 - 2|x|/a$
EQUIPHASE
$E_L(u) = \left[\Lambda_{\frac{1}{2}}(\pi au/2)\right]^2$
3-dB WIDTH = $\frac{1.28\lambda}{a} = \frac{73.3°}{a/\lambda}$
1.0
0.8
0.6
$E_L(u)$
0.4
0.2
-7.5 dB
-VE
-26.5 dB
au
3
2
1
0
1
2
3
au
0.6
0.5
0.4
0.3
0.2
Full Sine
$F(x) = \sin(2\pi x/a)$
$\bar{E}_L(u) = \frac{\pi \sin \pi au}{\pi^2 - (\pi au)^2}$
0.521
$\bar{E}_L(u)$
0.318
Antiphase Gable
$\bar{E}_L(u) = \frac{1}{\pi au}\left[1 - \Lambda_{\frac{1}{2}}(\pi au)\right]$
au
3
2
1
0
1
2
3

Figure 16

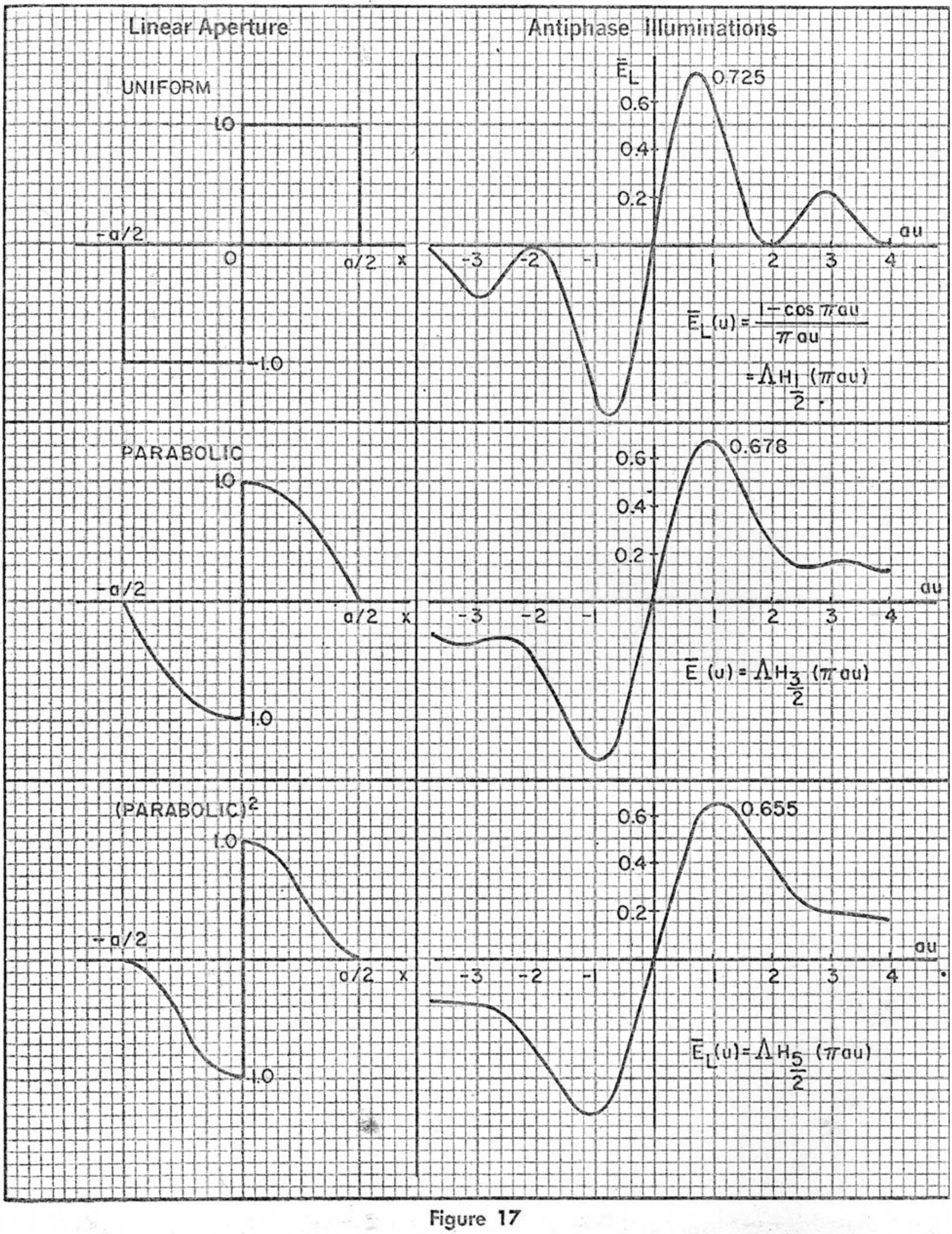

Linear Aperture
Antiphase Illuminations
UNIFORM
1.0
-a/2
0
a/2
x
-1.0
$\bar{E}_L$
0.725
0.6
0.4
0.2
au
-3
-2
-1
1
2
3
4
$\bar{E}_L(u) = \frac{1-\cos \pi au}{\pi au}$
$= \Lambda H_{\frac{1}{2}} (\pi au)$
PARABOLIC
1.0
-a/2
a/2
x
1.0
0.678
0.6
0.4
0.2
au
-3
-2
-1
1
2
3
4
$\bar{E}(u) = \Lambda H_{\frac{3}{2}} (\pi au)$
(PARABOLIC)^2
1.0
-a/2
a/2
x
1.0
0.655
0.6
0.4
0.2
au
-3
-2
-1
1
2
3
4
$\bar{E}_L(u) = \Lambda H_{\frac{5}{2}} (\pi au)$

Figure 17

√

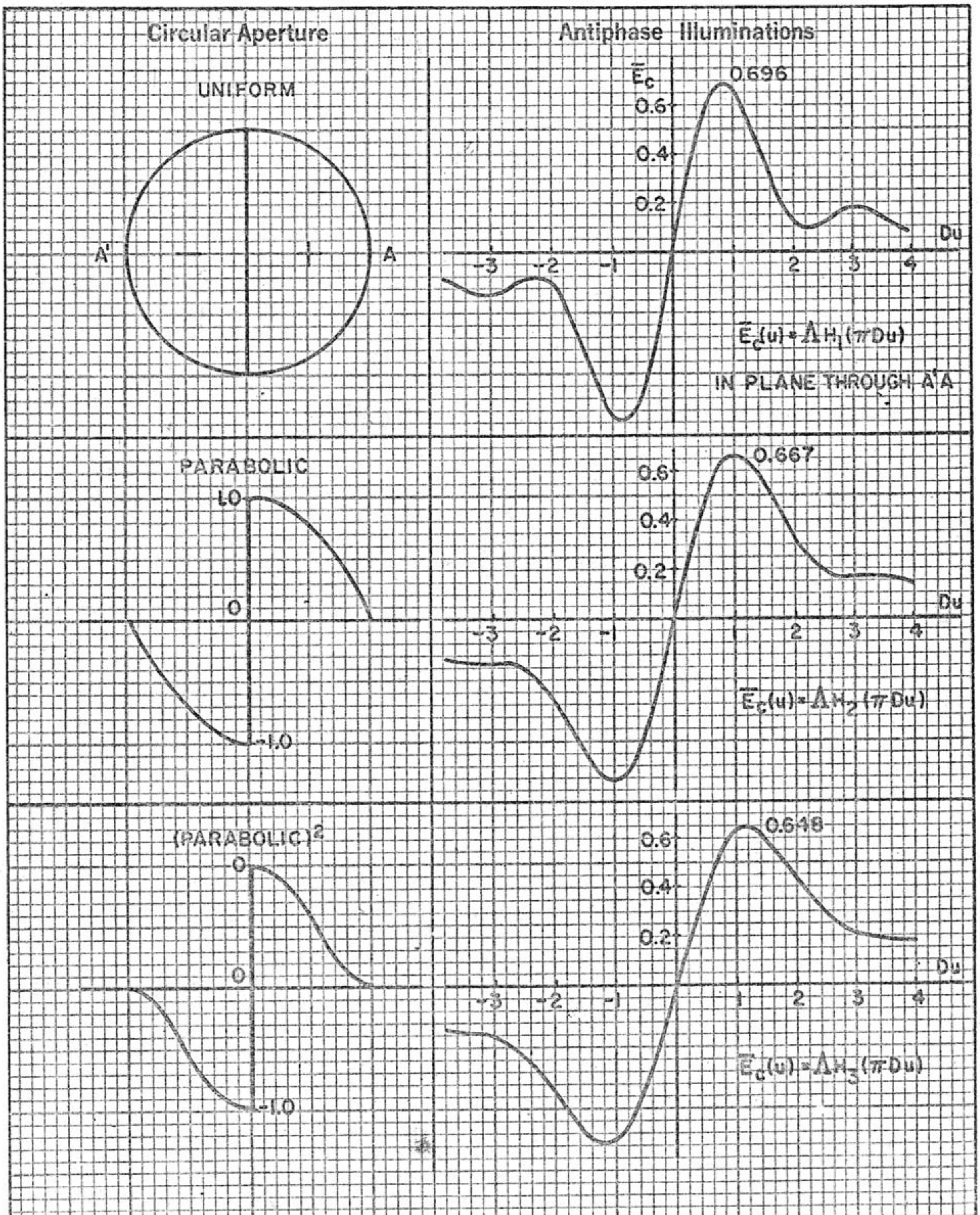

Circular Aperture
Antiphase Illuminations
UNIFORM
A'
A
$\bar{E}_c$
0.696
0.6
0.4
0.2
Du
-3
-2
-1
2
3
4
$\bar{E}_c(u) = \Lambda H_1(\pi Du)$
IN PLANE THROUGH A'A
PARABOLIC
1.0
0
-1.0
0.667
0.6
0.4
0.2
Du
-3
-2
-1
2
3
4
$\bar{E}_c(u) = \Lambda H_2(\pi Du)$
(PARABOLIC)²
0
0
-1.0
0.648
0.6
0.4
0.2
Du
-3
-2
-1
1
2
3
4
$\bar{E}_c(u) = \Lambda H_3(\pi Du)$

Figure 18

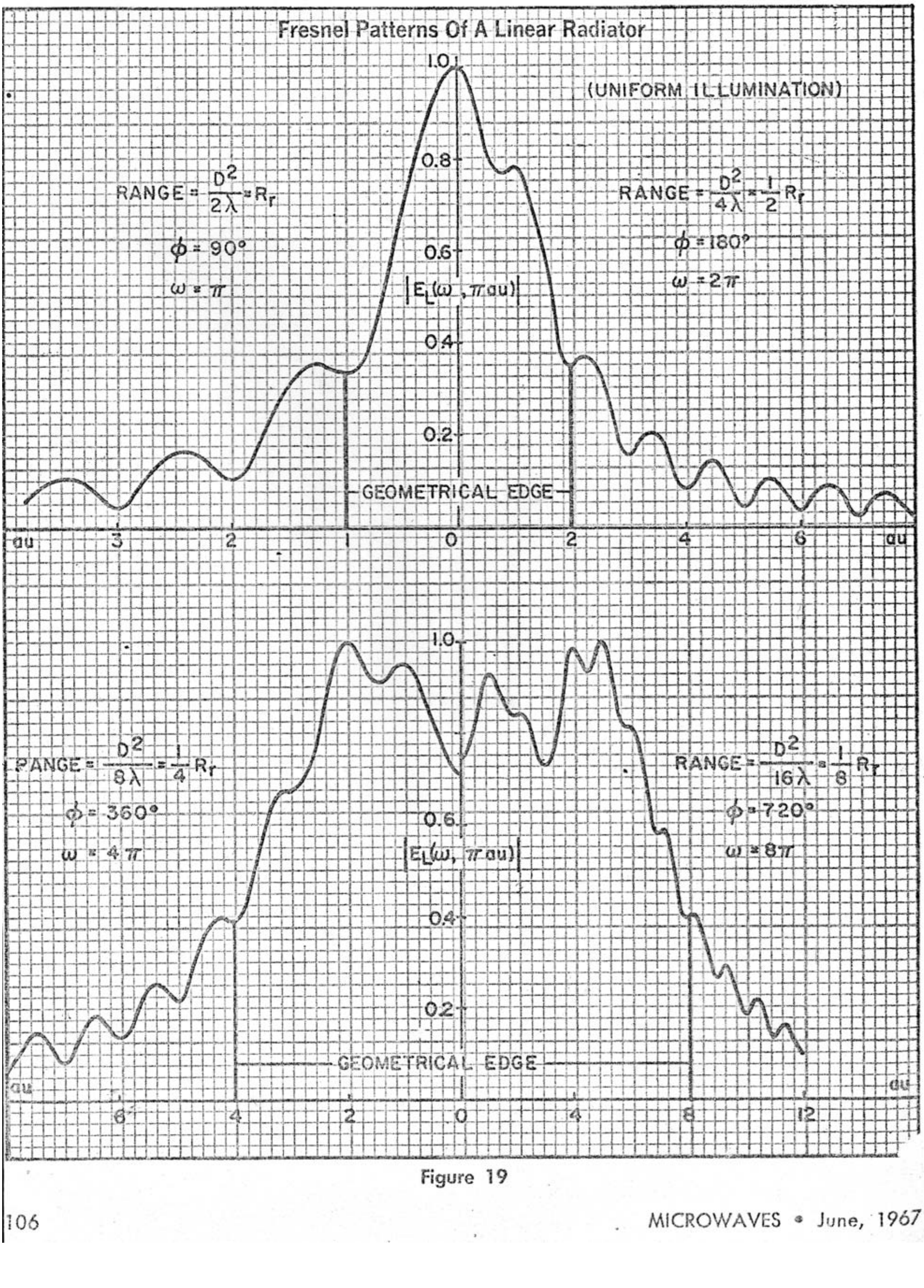
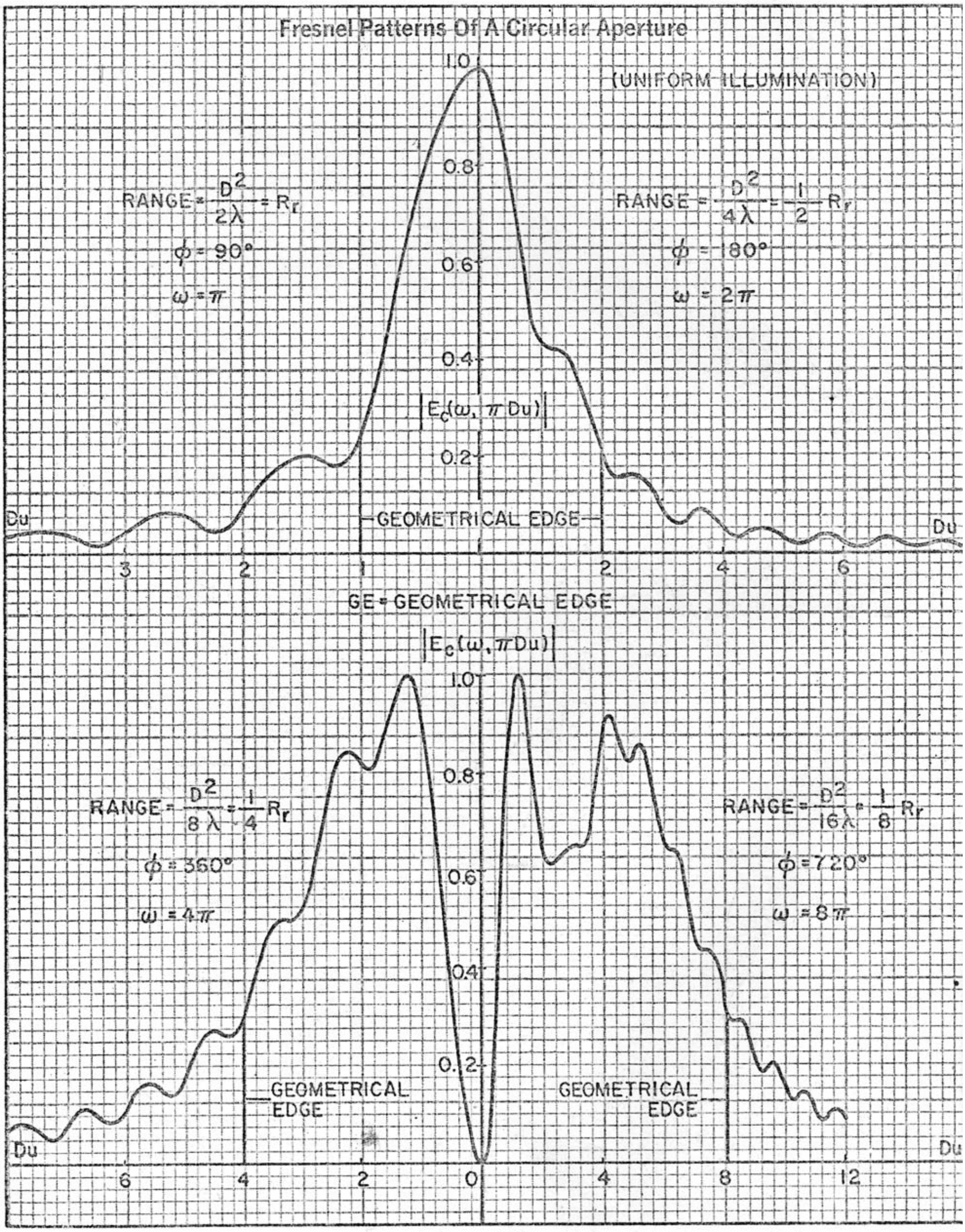

Fresnel Patterns Of A Linear Radiator
(UNIFORM ILLUMINATION)
RANGE = D2/2λ = Rr
φ = 90°
ω = π
RANGE = D2/4λ = 1/2 Rr
φ = 180°
ω = 2π
|EL(ω, π au)|
1.0
0.8
0.6
0.4
0.2
GEOMETRICAL EDGE
au
3
2
1
0
2
4
6
au
RANGE = D2/8λ = 1/4 Rr
φ = 360°
ω = 4π
RANGE = D2/16λ = 1/8 Rr
φ = 720°
ω = 8π
|EL(ω, π au)|
1.0
0.6
0.4
0.2
GEOMETRICAL EDGE
au
6
4
2
0
4
8
12
au

Figure 19



Fresnel Patterns Of A Circular Aperture
(UNIFORM ILLUMINATION)
RANGE = D²/2λ = Rr
φ = 90°
ω = π
RANGE = D²/4λ = 1/2 Rr
φ = 180°
ω = 2π
|Ec(ω, πDu)|
GEOMETRICAL EDGE
Du
GE = GEOMETRICAL EDGE
|Ec(ω, πDu)|
RANGE = D²/8λ = 1/4 Rr
φ = 360°
ω = 4π
RANGE = D²/16λ = 1/8 Rr
φ = 720°
ω = 8π
GEOMETRICAL EDGE
GEOMETRICAL EDGE
Du

Figure 20